CONSENSUS: BRIDGING THEORY AND PRACTICE

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I certify that I have read this dissertation and that, in my opinion, it is fully adequate in scope and quality as a dissertation for the degree of Doctor of Philosophy.

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Abstract

Distributed consensus is fundamental to building fault-tolerant systems. It allows a collection of machines to work as a coherent group that can survive the failures of some of its members. Unfortunately, the most common consensus algorithm, Paxos, is widely regarded as difficult to understand and implement correctly.

This dissertation presents a new consensus algorithm called Raft, which was designed for understandability. Raft first elects a server as leader, then concentrates all decision-making onto the leader. These two basic steps are relatively independent and form a better structure than Paxos, whose components are hard to separate. Raft elects a leader using voting and randomized timeouts. The election guarantees that the leader already stores all the information it needs, so data only flows outwards from the leader to other servers. Compared to other leader-based algorithms, this reduces mechanism and simplifies the behavior. Once a leader is elected, it manages a replicated log. Raft leverages a simple invariant on how logs grow to reduce the algorithm’s state space and accomplish this task with minimal mechanism.

Raft is also more suitable than previous algorithms for real-world implementations. It performs well enough for practical deployments, and it addresses all aspects of building a complete system, including how to manage client interactions, how to change the cluster membership, and how to compact the log when it grows too large. To change the cluster membership, Raft allows adding or removing one server at a time (complex changes can be composed from these basic steps), and the cluster continues servicing requests throughout the change.

We believe that Raft is superior to Paxos and other consensus algorithms, both for educational purposes and as a foundation for implementation. Results from a user study demonstrate that Raft is easier for students to learn than Paxos. The algorithm has been formally specified and proven, its leader election algorithm works well in a variety of environments, and its performance is equivalent to Multi-Paxos. Many implementations of Raft are now available, and several companies are deploying Raft.
Preface

This dissertation expands on a paper written by Diego Ongaro and John Ousterhout entitled *In Search of an Understandable Consensus Algorithm* [89]. Most of the paper’s content is included in some form in this dissertation. It is reproduced in this dissertation and licensed under the Creative Commons Attribution license with permission from John Ousterhout.

Readers may want to refer to the Raft website [92] for videos about Raft and an interactive visualization of Raft.
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Thanks to my family and friends for supporting me throughout the ups and downs of grad school. Mom, thanks for continuously pushing me to do well academically, even when I didn’t see the point. I still don’t know how you got me out of bed at 6 a.m. all those mornings. Dad, thanks for helping us earn these six (seven?) degrees, and I hope we’ve made you proud. Zeide, I wish I could give you a copy of this small book for your collection. Ernesto, thanks for sparking my interest in computers; I still think they’re pretty cool. Laura, I’ll let you know if and when I discover a RAMCloud. Thanks for listening to hours of my drama, even when you didn’t understand the nouns. Jenny, thanks for helping me get through the drudgery of writing this dissertation and for making me smile the whole way through. You’re crazy for having wanted to read this, and you’re weird for having enjoyed it.

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Thanks to the many professors who have advised me along the way. John Ousterhout, my Ph.D.
advisor, should be a coauthor on this dissertation (but I don’t think they would give me a degree that way). I have never learned as much professionally from any other person. John teaches by setting a great example of how to code, to evaluate, to design, to think, and to write well. I have never quite been on David Mazieres’s same wavelength; he’s usually 10–30 minutes ahead in conversation. As soon as I could almost keep up with him regarding consensus, he moved on to harder Byzantine consensus problems. Nevertheless, David has looked out for me throughout my years in grad school, and I’ve picked up some of his passion for building useful systems and, more importantly, having fun doing so. Mendel Rosenblum carries intimate knowledge of low level details like x86 instruction set, yet also manages to keep track of the big picture. He’s helped me with both over the years, surprising me with how quickly he can solve my technical problems and how clear my predicaments are when put into his own words. Thanks to Christos Kozyrakis and Stephen Weitzman for serving on my defense committee, and thanks to Alan Cox and Scott Rixner for introducing me to research during my undergraduate studies at Rice.

Many people contributed directly to this dissertation work. A special thanks goes to David Mazieres and Ezra Hoch for each finding a bug in earlier versions of Raft. David emailed us one night at 2:45 a.m. as he was reading through the Raft lecture slides for the user study. He wrote that he found “one thing quite hard to follow in the slides,” which turned out to be a major issue in Raft’s safety. Ezra found a liveness bug in membership changes. He posted to the Raft mailing list, “What if the following happens?” [35], and described an unfortunate series of events that could leave a cluster unable to elect a leader. Thanks also to Hugues Evrard for finding a small omission in the formal specification.

The user study would not have been possible without the support of Ali Ghodsi, David Mazieres, and the students of CS 294-91 at Berkeley and CS 240 at Stanford. Scott Klemmer helped us design the user study, and Nelson Ray advised us on statistical analysis. The Paxos slides for the user study borrowed heavily from a slide deck originally created by Lorenzo Alvisi.

Many people provided feedback on other content in this dissertation. In addition to my reading committee, Jennifer Wolochow provided helpful comments on the entire dissertation. Blake Mizerany, Xiang Li, and Yicheng Qin at CoreOS pushed me to simplify the membership change algorithm towards single-server changes. Anirban Rahut from Splunk pointed out that membership changes may be needlessly slow when a server joins with an empty log. Laura Ongaro offered helpful feedback on the user study chapter. Asaf Cidon helped direct me in finding the probability of split votes during elections. Eddie Kohler helped clarify the trade-offs in Raft’s commitment rule, and Maciej Smolenski pointed out that because of it, if a leader were to restart an unbounded number of
times before it could mark entries committed, its log could grow without bound (see Chapter 11). Alexander Shraer helped clarify how membership changes work in Zab.

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Chapter 1

Introduction

Today’s datacenter systems and applications run in highly dynamic environments. They scale out by leveraging the resources of additional servers, and they grow and shrink according to demand. Server and network failures are also commonplace: about 2–4% of disk drives fail each year [103], servers crash about as often [22], and tens of network links fail every day in modern datacenters [31].

As a result, systems must deal with servers coming and going during normal operations. They must react to changes and adapt automatically within seconds; outages that are noticeable to humans are typically not acceptable. This is a major challenge in today’s systems; failure handling, coordination, service discovery, and configuration management are all difficult in such dynamic environments.

Fortunately, distributed consensus can help with these challenges. Consensus allows a collection of machines to work as a coherent group that can survive the failures of some of its members. Within a consensus group, failures are handled in a principled and proven way. Because consensus groups are highly available and reliable, other system components can use a consensus group as the foundation for their own fault tolerance. Thus, consensus plays a key role in building reliable large-scale software systems.

When we started this work, the need for consensus was becoming clear, but many systems still struggled with problems that consensus could solve. Some large-scale systems were still limited by a single coordination server as a single point of failure (e.g., HDFS [81, 2]). Many others included ad hoc replication algorithms that handled failures unsafely (e.g., MongoDB and Redis [44]). New systems had few options for readily available consensus implementations (ZooKeeper [38] was the most popular), forcing systems builders to conform to one or build their own.

Those choosing to implement consensus themselves usually turned to Paxos [48, 49]. Paxos had
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dominated the discussion of consensus algorithms over the last two decades: most implementations of consensus were based on Paxos or influenced by it, and Paxos had become the primary vehicle used to teach students about consensus.

Unfortunately, Paxos is quite difficult to understand, in spite of numerous attempts to make it more approachable. Furthermore, its architecture requires complex changes to support practical systems, and building a complete system based on Paxos requires developing several extensions for which the details have not been published or agreed upon. As a result, both system builders and students struggle with Paxos.

The two other well-known consensus algorithms are Viewstamped Replication [83, 82, 66] and Zab [42], the algorithm used in ZooKeeper. Although we believe both of these algorithms are incidentally better in structure that Paxos for building systems, neither has explicitly made this argument; they were not designed with simplicity or understandability as a primary goal. The burden of understanding and implementing these algorithms is still too high.

Each of these consensus options was difficult to understand and difficult to implement. Unfortunately, when the cost of implementing consensus with proven algorithms was too high, systems builders were left with a tough decision. They could avoid consensus altogether, sacrificing the fault tolerance or consistency of their systems, or they could develop their own ad hoc algorithm, often leading to unsafe behavior. Moreover, when the cost of explaining and understanding consensus was too high, not all instructors attempted to teach it, and not all students succeeded in learning it. Consensus is as fundamental as two-phase commit; ideally, as many students should learn it (even though consensus is fundamentally more difficult).

After struggling with Paxos ourselves, we set out to find a new consensus algorithm that could provide a better foundation for system building and education. Our approach was unusual in that our primary goal was understandability: could we define a consensus algorithm for practical systems and describe it in a way that is significantly easier to learn than Paxos? Furthermore, we wanted the algorithm to facilitate the development of intuitions that are essential for system builders. It was important not just for the algorithm to work, but for it to be obvious why it works.

This algorithm also had to be complete enough to address all aspects of building a practical system, and it had to perform well enough for practical deployments. The core algorithm not only had to specify the effects of receiving a message but also describe what should happen and when; these are equally important for systems builders. Similarly, it had to guarantee consistency, and it also had to provide availability whenever possible. It also had to address the many aspects of a system that go beyond reaching consensus, such as changing the members of the consensus group.
These are necessary in practice, and leaving this burden to systems builders would risk ad hoc, suboptimal, or even incorrect solutions.

The result of this work is a consensus algorithm called Raft. In designing Raft we applied specific techniques to improve understandability, including decomposition (Raft separates leader election, log replication, and safety) and state space reduction (Raft reduces the degree of nondeterminism and the ways servers can be inconsistent with each other). We also addressed all of the issues needed to build a complete consensus-based system. We considered each design choice carefully, not just for the benefit of our own implementation but also for the many others we hope to enable.

We believe that Raft is superior to Paxos and other consensus algorithms, both for educational purposes and as a foundation for implementation. It is simpler and more understandable than other algorithms; it is described completely enough to meet the needs of a practical system; it has several open-source implementations and is used by several companies; its safety properties have been formally specified and proven; and its efficiency is comparable to other algorithms.

The primary contributions of this dissertation are as follows:

- The design, implementation, and evaluation of the Raft consensus algorithm. Raft is similar in many ways to existing consensus algorithms (most notably, Oki and Liskov’s Viewstamped Replication [83, 66]), but it is designed for understandability. This led to several novel features. For example, Raft uses a stronger form of leadership than other consensus algorithms. This simplifies the management of the replicated log and makes Raft easier to understand.

- The evaluation of Raft’s understandability. A user study with 43 students at two universities shows that Raft is significantly easier to understand than Paxos: after learning both algorithms, 33 of these students were able to answer questions about Raft better than questions about Paxos. We believe this is the first scientific study to evaluate consensus algorithms based on teaching and learning.

- The design, implementation, and evaluation of Raft’s leader election mechanism. While many consensus algorithms do not prescribe a particular leader election algorithm, Raft includes a specific algorithm involving randomized timers. This adds only a small amount of mechanism to the heartbeats already required for any consensus algorithm, while resolving conflicts simply and rapidly. The evaluation of leader election investigates its behavior and performance, concluding that this simple approach is sufficient in a wide variety of practical environments. It typically elects a leader in under 20 times the cluster’s one-way network latency.
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- The design and implementation of Raft’s cluster membership change mechanism. Raft allows adding or removing a single server at a time; these operations preserve safety simply, since at least one server overlaps any majority during the change. More complex changes in membership are implemented as a series of single-server changes. Raft allows the cluster to continue operating normally during changes, and membership changes can be implemented with only a few extensions to the basic consensus algorithm.

- A thorough discussion and implementation of the other components necessary for a complete consensus-based system, including client interaction and log compaction. Although we do not believe these aspects of Raft to be particularly novel, a complete description is important for understandability and to enable others to build real systems. We have implemented a complete consensus-based service to explore and address all of the design decisions involved.

- A proof of safety and formal specification for the Raft algorithm. The level of precision in the formal specification aids in reasoning carefully about the algorithm and clarifying details in the algorithm’s informal description. The proof of safety helps build confidence in Raft’s correctness. It also aids others who wish to extend Raft by clarifying the implications for safety of their extensions.

We have implemented many of the designs in this dissertation in an open-source implementation of Raft called LogCabin [86]. LogCabin served as our test platform for new ideas in Raft and as a way to verify that we understood the issues of building a complete and practical system. The implementation is described in more detail in Chapter 10.

The remainder of this dissertation introduces the replicated state machine problem and discusses the strengths and weaknesses of Paxos (Chapter 2); presents the Raft consensus algorithm, its extensions for cluster membership changes and log compaction, and how clients interact with Raft (Chapters 3–6); evaluates Raft for understandability, correctness, and leader election and log replication performance (Chapters 7–10); and discusses related work (Chapter 11).
Chapter 2

Motivation

Consensus is a fundamental problem in fault-tolerant systems: how can servers reach agreement on shared state, even in the face of failures? This problem arises in a wide variety of systems that need to provide high levels of availability and cannot compromise on consistency; thus, consensus is used in virtually all consistent large-scale storage systems. Section 2.1 describes how consensus is typically used to create replicated state machines, a general-purpose building block for fault-tolerant systems; Section 2.2 discusses various ways replicated state machines are used in larger systems; and Section 2.3 discusses the problems with the Paxos consensus protocol, which Raft aims to address.

2.1 Achieving fault tolerance with replicated state machines

Consensus algorithms typically arise in the context of replicated state machines [102]. In this approach, state machines on a collection of servers compute identical copies of the same state and can continue operating even if some of the servers are down. Replicated state machines are used to solve a variety of fault tolerance problems in distributed systems, as described in Section 2.2. Examples of replicated state machines include Chubby [11] and ZooKeeper [38], which both provide hierarchical key-value stores for small amounts of configuration data. In addition to basic operations such as get and put, they also provide synchronization primitives like compare-and-swap, enabling concurrent clients to coordinate safely.

Replicated state machines are typically implemented using a replicated log, as shown in Figure 2.1. Each server stores a log containing a series of commands, which its state machine executes in order. Each log contains the same commands in the same order, so each state machine processes
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Figure 2.1: Replicated state machine architecture. The consensus algorithm manages a replicated log containing state machine commands from clients. The state machines process identical sequences of commands from the logs, so they produce the same outputs.

the same sequence of commands. Since the state machines are deterministic, each computes the same state and the same sequence of outputs.

Keeping the replicated log consistent is the job of the consensus algorithm. The consensus module on a server receives commands from clients and adds them to its log. It communicates with the consensus modules on other servers to ensure that every log eventually contains the same requests in the same order, even if some servers fail. Once commands are properly replicated, they are said to be committed. Each server’s state machine processes committed commands in log order, and the outputs are returned to clients. As a result, the servers appear to form a single, highly reliable state machine.

Consensus algorithms for practical systems typically have the following properties:

• They ensure safety (never returning an incorrect result) under all non-Byzantine conditions, including network delays, partitions, and packet loss, duplication, and reordering.

• They are fully functional (available) as long as any majority of the servers are operational and can communicate with each other and with clients. Thus, a typical cluster of five servers can tolerate the failure of any two servers. Servers are assumed to fail by stopping; they may later recover from state on stable storage and rejoin the cluster.

• They do not depend on timing to ensure the consistency of the logs: faulty clocks and extreme message delays can, at worst, cause availability problems. That is, they maintain safety under an asynchronous model [71], in which messages and processors proceed at arbitrary speeds.
The nodes in the cluster coordinate among themselves by reading from and writing to the replicated state machine. One leader actively manages the nodes in the cluster and records its state using the replicated state machine. Other standby servers are passive until the leader fails.

Figure 2.2: Common patterns for using a single replicated state machine.

- In the common case, a command can complete as soon as a majority of the cluster has responded to a single round of remote procedure calls; a minority of slow servers need not impact overall system performance.

2.2 Common use cases for replicated state machines

Replicated state machines are a general-purpose building block for making systems fault-tolerant. They can be used in a variety of ways, and this section discusses some typical usage patterns.

Most common deployments of consensus have just three or five servers forming one replicated state machine. Other servers can then use this state machine to coordinate their activities, as shown in Figure 2.2(a). These systems often use the replicated state machine to provide group membership, configuration management, or locks [38]. As a more specific example, the replicated state machine could provide a fault-tolerant work queue, and other servers could coordinate using the replicated state machine to assign work to themselves.

A common simplification to this usage is shown in Figure 2.2(b). In this pattern, one server acts as leader, managing the rest of the servers. The leader stores its critical data in the consensus system. In case it fails, other standby servers compete for the position of leader, and if they succeed, they use the data in the consensus system to continue operations. Many large-scale storage systems that have a single cluster leader, such as GFS [30], HDFS [105], and RAMCloud [90], use this approach.

Consensus is also sometimes used to replicate very large amounts of data, as shown in Figure 2.3. Large storage systems, such as Megastore [5], Spanner [20], and Scatter [32], store too
Figure 2.3: Partitioned large-scale storage system using consensus. For scale, data is partitioned across many replicated state machines. Operations that span partitions use a two-phase commit protocol.

much data to fit in a single group of servers. They partition their data across many replicated state machines, and operations that span multiple partitions use a two-phase commit protocol (2PC) to maintain consistency.

2.3 What’s wrong with Paxos?

Over the last ten years, Leslie Lamport’s Paxos protocol [48] has become almost synonymous with consensus: it is the protocol most commonly taught in courses, and most implementations of consensus use it as a starting point. Paxos first defines a protocol capable of reaching agreement on a single decision, such as a single replicated log entry. We refer to this subset as **single-decree Paxos**.

Paxos then combines multiple instances of this protocol to facilitate a series of decisions such as a log (**Multi-Paxos**). Single-decree Paxos is summarized in Figure 2.4, and Multi-Paxos is summarized in Figure A.5. Paxos ensures safety and liveness (it eventually reaches consensus, assuming an adequate failure detector is used to avoid proposer livelock), and its correctness has been proven. Multi-Paxos is efficient in the normal case, and Paxos supports changes in cluster membership [69].

Unfortunately, Paxos has two significant drawbacks. The first drawback is that Paxos is exceptionally difficult to understand. The full explanation [48] is notoriously opaque; few people succeed in understanding it, and only with great effort. As a result, there have been several attempts to explain Paxos in simpler terms [49, 60, 61]. These explanations focus on the single-decree subset, yet they are still challenging. In an informal survey of attendees at NSDI 2012, we found few people who were comfortable with Paxos, even among seasoned researchers. We struggled with Paxos ourselves; we were not able to understand the complete protocol until after reading several explanations and designing our own alternative protocol, a process that took almost a year.
We hypothesize that Paxos’ opaqueness stems from its choice of the single-decree subset as its foundation. Single-decree Paxos is dense and subtle: it is divided into two stages that do not have simple intuitive explanations and cannot be understood independently. Because of this, it is difficult to develop intuitions about why the single-decree protocol works. The composition rules for Multi-Paxos add significant additional complexity and subtlety. We believe that the overall problem of reaching consensus on multiple decisions (i.e., a log instead of a single entry) can be decomposed in other ways that are more direct and obvious.

The second problem with Paxos is that it does not provide a good foundation for building practical implementations. One reason is that there is no widely agreed-upon algorithm for Multi-Paxos. Lamport’s descriptions are mostly about single-decree Paxos; he sketched possible approaches to Multi-Paxos, but many details are missing. There have been several attempts to flesh out and optimize Paxos, such as [77], [108], and [46], but these differ from each other and from Lamport’s sketches. Systems such as Chubby [15] have implemented Paxos-like algorithms, but in most cases their details have not been published.

Furthermore, the Paxos architecture is a poor one for building practical systems; this is another consequence of the single-decree decomposition. For example, there is little benefit to choosing a collection of log entries independently and then melding them into a sequential log; this just adds complexity. It is simpler and more efficient to design a system around a log, where new entries are

---

**Figure 2.4:** Summary of the single-decree Paxos consensus protocol. See [49] for a detailed explanation.
appended sequentially in a constrained order. Another problem is that Paxos uses a symmetric peer-to-peer approach at its core (though it also suggests a weak form of leadership as a performance optimization). This makes sense in a simplified world where only one decision will be made, but few practical systems use this approach. If a series of decisions must be made, it is simpler and faster to first elect a leader, then have the leader coordinate the decisions. (Chapter 11 discusses Egalitarian Paxos, a recent variant of Paxos that does not use a leader but in some situations can be more efficient than algorithms that do; however, this algorithm is much more complex than leader-based algorithms.)

As a result, practical systems bear little resemblance to Paxos. Each implementation begins with Paxos, discovers the difficulties in implementing it, and then develops a significantly different architecture. This is time-consuming and error-prone, and the difficulties of understanding Paxos exacerbate the problem. Paxos’ formulation may be a good one for proving theorems about its correctness, but real implementations are so different from Paxos that the proofs have little value. The following comment from the Chubby implementers is typical:

There are significant gaps between the description of the Paxos algorithm and the needs of a real-world system. . . . the final system will be based on an unproven protocol [15].

Because of these problems, we concluded that Paxos does not provide a good foundation either for system building or for education. Given the importance of consensus in large-scale software systems, we decided to see if we could design an alternative consensus algorithm with better properties than Paxos. Raft is the result of that experiment.
Chapter 3

Basic Raft algorithm

This chapter presents the Raft algorithm. We designed Raft to be as understandable as possible; the first section describes our approach to designing for understandability. The following sections describe the algorithm itself and include examples of design choices we made for understandability.

3.1 Designing for understandability

We had several goals in designing Raft: it must provide a complete and practical foundation for system building, so that it significantly reduces the amount of design work required of developers; it must be safe under all conditions and available under typical operating conditions; and it must be efficient for common operations. But our most important goal—and most difficult challenge—was understandability. It must be possible for a large audience to understand the algorithm comfortably. In addition, it must be possible to develop intuitions about the algorithm, so that system builders can make the extensions that are inevitable in real-world implementations.

There were numerous points in the design of Raft where we had to choose among alternative approaches. In these situations we evaluated the alternatives based on understandability: how hard is it to explain each alternative (for example, how complex is its state space, and does it have subtle implications?), and how easy will it be for a reader to completely understand the approach and its implications?

We recognize that there is a high degree of subjectivity in such analysis; nonetheless, we used two techniques that are generally applicable. The first technique is the well-known approach of problem decomposition: wherever possible, we divided problems into separate pieces that could be solved, explained, and understood relatively independently. For example, in Raft we separated
CHAPTER 3. BASIC RAFT ALGORITHM

leader election, log replication, and safety.

Our second approach was to simplify the state space by reducing the number of states to consider, making the system more coherent and eliminating nondeterminism where possible. Specifically, logs are not allowed to have holes, and Raft limits the ways in which logs can become inconsistent with each other. Although in most cases we tried to eliminate nondeterminism, there are some situations where nondeterminism actually improves understandability. In particular, randomized approaches introduce nondeterminism, but they tend to reduce the state space by handling all possible choices in a similar fashion (“choose any; it doesn’t matter”). We used randomization to simplify the Raft leader election algorithm.

3.2 Raft overview

Raft is an algorithm for managing a replicated log of the form described in Section 2.1. Figure 3.1 summarizes the algorithm in condensed form for reference, and Figure 3.2 lists key properties of the algorithm; the elements of these figures are discussed piecewise over the rest of this chapter.

Raft implements consensus by first electing a server as leader, then giving the leader complete responsibility for managing the replicated log. The leader accepts log entries from clients, replicates them on other servers, and tells servers when it is safe to apply log entries to their state machines. Having a leader simplifies the management of the replicated log. For example, the leader can decide where to place new entries in the log without consulting other servers, and data flows in a simple fashion from the leader to other servers. A leader can fail or become disconnected from the other servers, in which case a new leader is elected.

Given the leader approach, Raft decomposes the consensus problem into three relatively independent subproblems, which are discussed in the subsections that follow:

- **Leader election**: a new leader must be chosen when starting the cluster and when an existing leader fails (Section 3.4).

- **Log replication**: the leader must accept log entries from clients and replicate them across the cluster, forcing the other logs to agree with its own (Section 3.5).

- **Safety**: the key safety property for Raft is the State Machine Safety Property in Figure 3.2: if any server has applied a particular log entry to its state machine, then no other server may apply a different command for the same log index. Section 3.6 describes how Raft ensures this
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#### State

<table>
<thead>
<tr>
<th>Persistent state on all servers: (Updated on stable storage before responding to RPCs)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>currentTerm</strong></td>
</tr>
<tr>
<td><strong>votedFor</strong></td>
</tr>
<tr>
<td><strong>log[]</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volatile state on all servers:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>commitIndex</strong></td>
</tr>
<tr>
<td><strong>lastApplied</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volatile state on leaders: (Reinitialized after election)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>nextIndex[]</strong></td>
</tr>
<tr>
<td><strong>matchIndex[]</strong></td>
</tr>
</tbody>
</table>

#### RequestVote RPC

Invoked by candidates to gather votes (§3.4).

<table>
<thead>
<tr>
<th>Arguments:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>term</strong></td>
</tr>
<tr>
<td><strong>candidateId</strong></td>
</tr>
<tr>
<td><strong>lastLogIndex</strong></td>
</tr>
<tr>
<td><strong>lastLogTerm</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>term</strong></td>
</tr>
<tr>
<td><strong>voteGranted</strong></td>
</tr>
</tbody>
</table>

**Receiver implementation:**
1. Reply false if term < currentTerm (§3.3).
2. If votedFor is null or candidateId, and candidate’s log is at least as up-to-date as receiver’s log, grant vote (§3.4, §3.6)

#### Rules for Servers

<table>
<thead>
<tr>
<th>All Servers:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• If commitIndex &gt; lastApplied: increment lastApplied, apply log[lastApplied] to state machine (§3.5)</td>
</tr>
<tr>
<td>• If RPC request or response contains term T &gt; currentTerm: set currentTerm = T, convert to follower (§3.3)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Followers (§3.4):</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Respond to RPCs from candidates and leaders</td>
</tr>
<tr>
<td>• If election timeout elapses without receiving AppendEntries RPC from current leader or granting vote to candidate: convert to candidate</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Candidates (§3.4):</th>
</tr>
</thead>
<tbody>
<tr>
<td>• On conversion to candidate, start election:</td>
</tr>
<tr>
<td>• Increment currentTerm</td>
</tr>
<tr>
<td>• Vote for self</td>
</tr>
<tr>
<td>• Reset election timer</td>
</tr>
<tr>
<td>• Send RequestVote RPCs to all other servers</td>
</tr>
<tr>
<td>• If votes received from majority of servers: become leader</td>
</tr>
<tr>
<td>• If AppendEntries RPC received from new leader: convert to follower</td>
</tr>
<tr>
<td>• If election timeout elapses: start new election</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Leaders:</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Upon election: send initial empty AppendEntries RPC (heartbeat) to each server; repeat during idle periods to prevent election timeouts (§3.4)</td>
</tr>
<tr>
<td>• If command received from client: append entry to local log, respond after entry applied to state machine (§3.5)</td>
</tr>
<tr>
<td>• If last log index ≥ nextIndex for a follower: send AppendEntries RPC with log entries starting at nextIndex</td>
</tr>
<tr>
<td>• If successful: update nextIndex and matchIndex for follower (§3.5)</td>
</tr>
<tr>
<td>• If AppendEntries fails because of log inconsistency: decrement nextIndex and retry (§3.5)</td>
</tr>
<tr>
<td>• If there exists an N such that N &gt; commitIndex, a majority of matchIndex[i] ≥ N, and log[N].term = currentTerm: set commitIndex = N (§3.5, §3.6).</td>
</tr>
</tbody>
</table>

#### AppendEntries RPC

Invoked by leader to replicate log entries (§3.5); also used as heartbeat (§3.4).

<table>
<thead>
<tr>
<th>Arguments:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>term</strong></td>
</tr>
<tr>
<td><strong>leaderId</strong></td>
</tr>
<tr>
<td><strong>prevLogIndex</strong></td>
</tr>
<tr>
<td><strong>prevLogTerm</strong></td>
</tr>
<tr>
<td><strong>entries[]</strong></td>
</tr>
<tr>
<td><strong>leaderCommit</strong></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Results:</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>term</strong></td>
</tr>
<tr>
<td><strong>success</strong></td>
</tr>
</tbody>
</table>

**Receiver implementation:**
1. Reply false if term < currentTerm (§3.3).
2. Reply false if log doesn’t contain an entry at prevLogIndex whose term matches prevLogTerm (§3.5).
3. If an existing entry conflicts with a new one (same index but different terms), delete the existing entry and all that follow it (§3.5).
4. Append any new entries not already in the log |
5. If leaderCommit > commitIndex, set commitIndex = min(leaderCommit, index of last new entry)

---

**Figure 3.1:** A condensed summary of the Raft consensus algorithm (excluding membership changes, log compaction, and client interaction). The server behavior in the lower-right box is described as a set of rules that trigger independently and repeatedly. Section numbers such as §3.4 indicate where particular features are discussed. The formal specification in Appendix B describes the algorithm more precisely.
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Election Safety
At most one leader can be elected in a given term. §3.4

Leader Append-Only
A leader never overwrites or deletes entries in its log; it only appends new entries. §3.5

Log Matching
If two logs contain an entry with the same index and term, then the logs are identical in all entries up through the given index. §3.5

Leader Completeness
If a log entry is committed in a given term, then that entry will be present in the logs of the leaders for all higher-numbered terms. §3.6

State Machine Safety
If a server has applied a log entry at a given index to its state machine, no other server will ever apply a different log entry for the same index. §3.6.3

Figure 3.2: Raft guarantees that each of these properties is true at all times. The section numbers indicate where each property is discussed.

property; the solution involves an additional restriction on the election mechanism described in Section 3.4.

After presenting the consensus algorithm, this chapter discusses the issue of availability and the role of timing in the system (Section 3.9), and an optional extension to transfer leadership between servers (Section 3.10).

3.3 Raft basics

A Raft cluster contains several servers; five is a typical number, which allows the system to tolerate two failures. At any given time each server is in one of three states: leader, follower, or candidate. In normal operation there is exactly one leader and all of the other servers are followers. Followers are passive: they issue no requests on their own but simply respond to requests from leaders and candidates. The leader handles all client requests (if a client contacts a follower, the follower redirects it to the leader). The third state, candidate, is used to elect a new leader as described in Section 3.4. Figure 3.3 shows the states and their transitions; the transitions are discussed below.

Raft divides time into terms of arbitrary length, as shown in Figure 3.4. Terms are numbered with consecutive integers. Each term begins with an election, in which one or more candidates attempt to become leader as described in Section 3.4. If a candidate wins the election, then it serves as leader for the rest of the term. In some situations an election will result in a split vote. In this case
Figure 3.3: Server states. Followers only respond to requests from other servers. If a follower receives no communication, it becomes a candidate and initiates an election. A candidate that receives votes from a majority of the full cluster becomes the new leader. Leaders typically operate until they fail.

Figure 3.4: Time is divided into terms, and each term begins with an election. After a successful election, a single leader manages the cluster until the end of the term. Some elections fail, in which case the term ends without choosing a leader. The transitions between terms may be observed at different times on different servers.

the term will end with no leader; a new term (with a new election) will begin shortly. Raft ensures that there is at most one leader in a given term.

Different servers may observe the transitions between terms at different times, and in some situations a server may not observe an election or even entire terms. Terms act as a logical clock [47] in Raft, and they allow servers to detect obsolete information such as stale leaders. Each server stores a current term number, which increases monotonically over time. Current terms are exchanged whenever servers communicate; if one server’s current term is smaller than the other’s, then it updates its current term to the larger value. If a candidate or leader discovers that its term is out of date, it immediately reverts to follower state. If a server receives a request with a stale term number, it rejects the request.
Raft servers communicate using remote procedure calls (RPCs), and the basic consensus algorithm requires only two types of RPCs between servers. RequestVote RPCs are initiated by candidates during elections (Section 3.4), and AppendEntries RPCs are initiated by leaders to replicate log entries and to provide a form of heartbeat (Section 3.5). Leadership transfer (Section 3.10) and the mechanisms described in subsequent chapters introduce additional RPCs beyond the two in the core consensus algorithm.

We chose to structure communication in Raft as RPCs to simplify its communication patterns. Each request type has a corresponding response type, which also serves as the request’s acknowledgment. Raft assumes RPC requests and responses may be lost in the network; it is the requester’s responsibility to retry the RPC if it does not receive a response in a timely manner. Servers issue RPCs in parallel for best performance, and Raft does not assume the network preserves ordering between RPCs.

### 3.4 Leader election

Raft uses a heartbeat mechanism to trigger leader election. When servers start up, they begin as followers. A server remains in follower state as long as it receives valid RPCs from a leader or candidate. Leaders send periodic heartbeats (AppendEntries RPCs that carry no log entries) to all followers in order to maintain their authority. If a follower receives no communication over a period of time called the *election timeout*, then it assumes there is no viable leader and begins an election to choose a new leader.

To begin an election, a follower increments its current term and transitions to candidate state. It then votes for itself and issues RequestVote RPCs in parallel to each of the other servers in the cluster. A candidate continues in this state until one of three things happens: (a) it wins the election, (b) another server establishes itself as leader, or (c) another election timeout goes by with no winner. These outcomes are discussed separately in the paragraphs below.

A candidate wins an election if it receives votes from a majority of the servers in the full cluster for the same term. Each server will vote for at most one candidate in a given term, on a first-come-first-served basis (note: Section 3.6 adds an additional restriction on votes). The majority rule ensures that at most one candidate can win the election for a particular term (the Election Safety Property in Figure 3.2). Once a candidate wins an election, it becomes leader. It then sends heartbeat messages to all of the other servers to establish its authority and prevent new elections.

While waiting for votes, a candidate may receive an AppendEntries RPC from another server
claiming to be leader. If the leader’s term (included in its RPC) is at least as large as the candidate’s current term, then the candidate recognizes the leader as legitimate and returns to follower state. If the term in the RPC is smaller than the candidate’s current term, then the candidate rejects the RPC and continues in candidate state.

The third possible outcome is that a candidate neither wins nor loses the election: if many followers become candidates at the same time, votes could be split so that no candidate obtains a majority. When this happens, each candidate will time out and start a new election by incrementing its term and initiating another round of RequestVote RPCs. However, without extra measures split votes could repeat indefinitely.

Raft uses randomized election timeouts to ensure that split votes are rare and that they are resolved quickly. To prevent split votes in the first place, election timeouts are chosen randomly from a fixed interval (e.g., 150–300 ms). This spreads out the servers so that in most cases only a single server will time out; it wins the election and sends heartbeats before any other servers time out. The same mechanism is used to handle split votes. Each candidate restarts its randomized election timeout at the start of an election, and it waits for that timeout to elapse before starting the next election; this reduces the likelihood of another split vote in the new election. Chapter 9 shows that this approach elects a leader rapidly.

Elections are an example of how understandability guided our choice between design alternatives. Initially we planned to use a ranking system: each candidate was assigned a unique rank, which was used to select between competing candidates. If a candidate discovered another candidate with higher rank, it would return to follower state so that the higher ranking candidate could more easily win the next election. We found that this approach created subtle issues around availability (a lower-ranked server might need to time out and become a candidate again if a higher-ranked server fails, but if it does so too soon, it can reset progress towards electing a leader). We made adjustments to the algorithm several times, but after each adjustment new corner cases appeared. Eventually we concluded that the randomized retry approach is more obvious and understandable.

3.5 Log replication

Once a leader has been elected, it begins servicing client requests. Each client request contains a command to be executed by the replicated state machine. The leader appends the command to its log as a new entry, then issues AppendEntries RPCs in parallel to each of the other servers to replicate the entry. When the entry has been safely replicated (as described below), the leader applies the
Figure 3.5: Logs are composed of entries, which are numbered sequentially. Each entry contains the term in which it was created (the number in each box) and a command for the state machine. An entry is considered *committed* if it is safe for that entry to be applied to state machines.

entry to its state machine and returns the result of that execution to the client. If followers crash or run slowly, or if network packets are lost, the leader retries AppendEntries RPCs indefinitely (even after it has responded to the client) until all followers eventually store all log entries.

Logs are organized as shown in Figure 3.5. Each log entry stores a state machine command along with the term number when the entry was received by the leader. The term numbers in log entries are used to detect inconsistencies between logs and to ensure some of the properties in Figure 3.2. Each log entry also has an integer index identifying its position in the log.

The leader decides when it is safe to apply a log entry to the state machines; such an entry is called *committed*. Raft guarantees that committed entries are durable and will eventually be executed by all of the available state machines. A log entry is committed once the leader that created the entry has replicated it on a majority of the servers (e.g., entry 7 in Figure 3.5). This also commits all preceding entries in the leader’s log, including entries created by previous leaders. Section 3.6 discusses some subtleties when applying this rule after leader changes, and it also shows that this definition of commitment is safe. The leader keeps track of the highest index it knows to be committed, and it includes that index in future AppendEntries RPCs (including heartbeats) so that the other servers eventually find out. Once a follower learns that a log entry is committed, it applies the entry to its local state machine (in log order).
We designed the Raft log mechanism to maintain a high level of coherency between the logs on different servers. Not only does this simplify the system’s behavior and make it more predictable, but it is an important component of ensuring safety. Raft maintains the following properties, which together constitute the Log Matching Property in Figure 3.2:

- If two entries in different logs have the same index and term, then they store the same command.
- If two entries in different logs have the same index and term, then the logs are identical in all preceding entries.

The first property follows from the fact that a leader creates at most one entry with a given log index in a given term, and log entries never change their position in the log. The second property is guaranteed by a consistency check performed by AppendEntries. When sending an AppendEntries RPC, the leader includes the index and term of the entry in its log that immediately precedes the new entries. If the follower does not find an entry in its log with the same index and term, then it refuses the new entries. The consistency check acts as an induction step: the initial empty state of the logs satisfies the Log Matching Property, and the consistency check preserves the Log Matching Property whenever logs are extended. As a result, whenever AppendEntries returns successfully, the leader knows that the follower’s log is identical to its own log up through the new entries.

During normal operation, the logs of the leader and followers stay consistent, so the AppendEntries consistency check never fails. However, leader crashes can leave the logs inconsistent (the old leader may not have fully replicated all of the entries in its log). These inconsistencies can compound over a series of leader and follower crashes. Figure 3.6 illustrates the ways in which followers’ logs may differ from that of a new leader. A follower may be missing entries that are present on the leader, it may have extra entries that are not present on the leader, or both. Missing and extraneous entries in a log may span multiple terms.

In Raft, the leader handles inconsistencies by forcing the followers’ logs to duplicate its own. This means that conflicting entries in follower logs will be overwritten with entries from the leader’s log. Section 3.6 will show that this is safe when coupled with a restriction on elections.

To bring a follower’s log into consistency with its own, the leader must find the latest log entry where the two logs agree, delete any entries in the follower’s log after that point, and send the follower all of the leader’s entries after that point. All of these actions happen in response to the consistency check performed by AppendEntries RPCs. The leader maintains a nextIndex for each follower, which is the index of the next log entry the leader will send to that follower. When a leader first comes to power, it initializes all nextIndex values to the index just after the last one in
Figure 3.6: When the leader at the top comes to power, it is possible that any of scenarios (a–f) could occur in follower logs. Each box represents one log entry; the number in the box is its term. A follower may be missing entries (a–b), may have extra uncommitted entries (c–d), or both (e–f). For example, scenario (f) could occur if that server was the leader for term 2, added several entries to its log, then crashed before committing any of them; it restarted quickly, became leader for term 3, and added a few more entries to its log; before any of the entries in either term 2 or term 3 were committed, the server crashed again and remained down for several terms.
its log (11 in Figure 3.6). If a follower’s log is inconsistent with the leader’s, the AppendEntries consistency check will fail in the next AppendEntries RPC. After a rejection, the leader decrements the follower’s nextIndex and retries the AppendEntries RPC. Eventually the nextIndex will reach a point where the leader and follower logs match. When this happens, AppendEntries will succeed, which removes any conflicting entries in the follower’s log and appends entries from the leader’s log (if any). Once AppendEntries succeeds, the follower’s log is consistent with the leader’s, and it will remain that way for the rest of the term.

Until the leader has discovered where it and the follower’s logs match, the leader can send AppendEntries with no entries (like heartbeats) to save bandwidth. Then, once the matchIndex immediately precedes the nextIndex, the leader should begin to send the actual entries.

If desired, the protocol can be optimized to reduce the number of rejected AppendEntries RPCs. For example, when rejecting an AppendEntries request, the follower can include the term of the conflicting entry and the first index it stores for that term. With this information, the leader can decrement nextIndex to bypass all of the conflicting entries in that term; one AppendEntries RPC will be required for each term with conflicting entries, rather than one RPC per entry. Alternatively, the leader can use a binary search approach to find the first entry where the follower’s log differs from its own; this has better worst-case behavior. In practice, we doubt these optimizations are necessary, since failures happen infrequently and it is unlikely that there will be many inconsistent entries.

With this mechanism, a leader does not need to take any special actions to restore log consistency when it comes to power. It just begins normal operation, and the logs automatically converge in response to failures of the AppendEntries consistency check. A leader never overwrites or deletes entries in its own log (the Leader Append-Only Property in Figure 3.2).

This log replication mechanism exhibits the desirable consensus properties described in Section 2.1: Raft can accept, replicate, and apply new log entries as long as a majority of the servers are up; in the normal case a new entry can be replicated with a single round of RPCs to a majority of the cluster; and a single slow follower will not impact performance. The log replication algorithm is also practical to implement, since AppendEntries requests are manageable in size (leaders never need to send more than one entry in a single AppendEntries request to make progress). Some other consensus algorithms are described as sending entire logs over the network; this places a burden on the implementer to develop optimizations required for a practical implementation.
3.6 Safety

The previous sections described how Raft elects leaders and replicates log entries. However, the mechanisms described so far are not quite sufficient to ensure that each state machine executes exactly the same commands in the same order. For example, a follower might be unavailable while the leader commits several log entries, then it could be elected leader and overwrite these entries with new ones; as a result, different state machines might execute different command sequences.

This section completes the Raft algorithm by adding a restriction on which servers may be elected leader. The restriction ensures that the leader for any given term contains all of the entries committed in previous terms (the Leader Completeness Property from Figure 3.2). Given the election restriction, we then make the rules for commitment more precise. Finally, we present a proof sketch for the Leader Completeness Property and show how it leads to correct behavior of the replicated state machine.

3.6.1 Election restriction

In any leader-based consensus algorithm, the leader must eventually store all of the committed log entries. In some consensus algorithms, such as Viewstamped Replication [66], a leader can be elected even if it doesn’t initially contain all of the committed entries. These algorithms contain additional mechanisms to identify the missing entries and transmit them to the new leader, either during the election process or shortly afterwards. Unfortunately, this results in considerable additional mechanism and complexity. Raft uses a simpler approach where it guarantees that all the committed entries from previous terms are present on each new leader from the moment of its election, without the need to transfer those entries to the leader. This means that log entries only flow in one direction, from leaders to followers, and leaders never overwrite existing entries in their logs.

Raft uses the voting process to prevent a candidate from winning an election unless its log contains all committed entries. A candidate must contact a majority of the cluster in order to be elected, which means that every committed entry must be present in at least one of those servers. If the candidate’s log is at least as up-to-date as any other log in that majority (where “up-to-date” is defined precisely below), then it will hold all the committed entries. The RequestVote RPC implements this restriction: the RPC includes information about the candidate’s log, and the voter denies its vote if its own log is more up-to-date than that of the candidate.

Raft determines which of two logs is more up-to-date by comparing the index and term of the last entries in the logs. If the logs have last entries with different terms, then the log with the later
Figure 3.7: A time sequence showing why a leader cannot determine commitment using log entries from older terms. In (a) S1 is leader and partially replicates the log entry at index 2. In (b) S1 crashes; S5 is elected leader for term 3 with votes from S3, S4, and itself, and accepts a different entry at log index 2. In (c) S5 crashes; S1 restarts, is elected leader, and continues replication. At this point, the log entry from term 2 has been replicated on a majority of the servers, but it is not committed. If S1 crashes as in (d1), S5 could be elected leader (with votes from S2, S3, and S4) and overwrite the entry with its own entry from term 3. However, if S1 replicates an entry from its current term on a majority of the servers before crashing, as in (d2), then this entry is committed (S5 cannot win an election). At this point all preceding entries in the log are committed as well.
term is more up-to-date. If the logs end with the same term, then whichever log is longer is more up-to-date.

### 3.6.2 Committing entries from previous terms

As described in Section 3.5, a leader knows that an entry from its current term is committed once that entry is stored on a majority of the servers. If a leader crashes before committing an entry, future leaders will attempt to finish replicating the entry. However, a leader cannot immediately conclude that an entry from a previous term is committed once it is stored on a majority of servers. Figure 3.7 illustrates a situation where an old log entry is stored on a majority of servers, yet can still be overwritten by a future leader.

To eliminate problems like the one in Figure 3.7, Raft never commits log entries from previous terms by counting replicas. Only log entries from the leader’s current term are committed by counting replicas; once an entry from the current term has been committed in this way, then all prior entries are committed indirectly because of the Log Matching Property. There are some situations where a leader could safely conclude that an older log entry is committed (for example, if that entry is stored on every server), but Raft takes a more conservative approach for simplicity.

Raft incurs this extra complexity in the commitment rules because log entries retain their original term numbers when a leader replicates entries from previous terms. In other consensus algorithms, if a new leader re-replicates entries from prior “terms”, it must do so with its new “term number”. Raft’s approach makes it easier to reason about log entries, since they maintain the same term number over time and across logs. In addition, new leaders in Raft send fewer log entries from previous terms than in other algorithms, since other algorithms must send redundant log entries to renumber them before they can be committed; however, this may not be very important in practice, since leader changes should be rare.

### 3.6.3 Safety argument

Given the complete Raft algorithm, we can now argue more precisely that the Leader Completeness Property holds (this argument is based on the safety proof; see Chapter 8). We assume that the Leader Completeness Property does not hold, then we prove a contradiction. Suppose the leader for term $T$ (leader$_T$) commits a log entry from its term, but that log entry is not stored by the leader of some future term. Consider the smallest term $U > T$ whose leader (leader$_U$) does not store the entry.

1. The committed entry must have been absent from leader$_U$’s log at the time of its election.
Figure 3.8: If S1 (leader for term T) commits a new log entry from its term, and S5 is elected leader for a later term U, then there must be at least one server (S3) that accepted the log entry and also voted for S5.

(leaders never delete or overwrite entries).

2. leader\(_T\) replicated the entry on a majority of the cluster, and leader\(_U\) received votes from a majority of the cluster. Thus, at least one server ("the voter") both accepted the entry from leader\(_T\) and voted for leader\(_U\), as shown in Figure 3.8. The voter is key to reaching a contradiction.

3. The voter must have accepted the committed entry from leader\(_T\) \textit{before} voting for leader\(_U\); otherwise it would have rejected the AppendEntries request from leader\(_T\) (its current term would have been higher than T).

4. The voter still stored the entry when it voted for leader\(_U\), since every intervening leader contained the entry (by assumption), leaders never remove entries, and followers only remove entries if they conflict with the leader.

5. The voter granted its vote to leader\(_U\), so leader\(_U\)’s log must have been as up-to-date as the voter’s. This leads to one of two contradictions.

6. First, if the voter and leader\(_U\) shared the same last log term, then leader\(_U\)’s log must have been at least as long as the voter’s, so its log contained every entry in the voter’s log. This is a contradiction, since the voter contained the committed entry and leader\(_U\) was assumed not to.

7. Otherwise, leader\(_U\)’s last log term must have been larger than the voter’s. Moreover, it was larger than T, since the voter’s last log term was at least T (it contains the committed entry from term T). The earlier leader that created leader\(_U\)’s last log entry must have contained the
committed entry in its log (by assumption). Then, by the Log Matching Property, leader_{U}’s log must also contain the committed entry, which is a contradiction.

8. This completes the contradiction. Thus, the leaders of all terms greater than T must contain all entries from term T that are committed in term T.

9. The Log Matching Property guarantees that future leaders will also contain entries that are committed indirectly, such as index 2 in Figure 3.7(d2).

Given the Leader Completeness Property, we can prove the State Machine Safety Property from Figure 3.2, which states that if a server has applied a log entry at a given index to its state machine, no other server will ever apply a different log entry for the same index. At the time a server applies a log entry to its state machine, its log must be identical to the leader’s log up through that entry, and the entry must be committed. Now consider the lowest term in which any server applies a given log index; the Leader Completeness Property guarantees that the leaders for all higher terms will store that same log entry, so servers that apply the index in later terms will apply the same value. Thus, the State Machine Safety Property holds.

Finally, Raft requires servers to apply entries in log index order. Combined with the State Machine Safety Property, this means that all servers will apply exactly the same set of log entries to their state machines, in the same order.

3.7 Follower and candidate crashes

Until this point we have focused on leader failures. Follower and candidate crashes are much simpler to handle than leader crashes, and they are both handled in the same way. If a follower or candidate crashes (or the network link between it and the leader fails), then future RequestVote and AppendEntries RPCs sent to it will fail. Raft handles these failures by retrying indefinitely; if the crashed server restarts, then the RPC will complete successfully. If a server crashes after completing an RPC but before responding, then it will receive the same RPC again after it restarts. Raft RPCs have the same effect if repeated, so this causes no harm. For example, if a follower receives an AppendEntries request that includes log entries already present in its log, it ignores those entries in the new request.
3.8 Persisted state and server restarts

Raft servers must persist enough information to stable storage to survive server restarts safely. In particular, each server persists its current term and vote; this is necessary to prevent the server from voting twice in the same term or replacing log entries from a newer leader with those from a deposed leader. Each server also persists new log entries before they are counted towards the entries’ commitment; this prevents committed entries from being lost or “uncommitted” when servers restart.

Other state variables are safe to lose on a restart, as they can all be recreated. The most interesting example is the commit index, which can safely be reinitialized to zero on a restart. Even if every server restarts at the same time, the commit index will only temporarily lag behind its true value. Once a leader is elected and is able to commit a new entry, its commit index will advance, and it will quickly propagate this commit index to its followers.

The state machine can either be volatile or persistent. A volatile state machine must be recovered after restarts by reapplying log entries (after applying the latest snapshot; see Chapter 5). A persistent state machine, however, has already applied most entries after a restart; to avoid reapplying them, its last applied index must also be persistent.

If a server loses any of its persistent state, it cannot safely rejoin the cluster with its prior identity. Such a server can usually be added back into the cluster with a new identity by invoking a cluster membership change (see Chapter 4). If a majority of the cluster loses its persistent state, however, log entries may be lost and progress on cluster membership changes will not be possible; to proceed, a system administrator would need to admit the possibility of data loss.

3.9 Timing and availability

One of our requirements for Raft is that safety must not depend on timing: the system must not produce incorrect results just because some event happens more quickly or slowly than expected. However, availability (the ability of the system to respond to clients in a timely manner) must inevitably depend on timing. For example, if message exchanges take longer than the typical time between server crashes, candidates will not stay up long enough to win an election; without a steady leader, Raft cannot make progress.

Leader election is the aspect of Raft where timing is most critical. Raft will be able to elect and maintain a steady leader when the system satisfies the following timing requirement:

\[
\text{broadcastTime} \ll \text{electionTimeout} \ll \text{MTBF}
\]
In this inequality broadcastTime is the average time it takes a server to send RPCs in parallel to every server in the cluster and receive their responses; electionTimeout is the election timeout described in Section 3.4; and MTBF is the mean (average) time between failures for a single server. The broadcast time should be an order of magnitude less than the election timeout so that leaders can reliably send the heartbeat messages required to keep followers from starting elections; given the randomized approach used for election timeouts, this inequality also makes split votes unlikely. The election timeout should be a few orders of magnitude less than MTBF so that the system makes steady progress. When the leader crashes, the system will be unavailable for roughly the election timeout; we would like this to represent only a small fraction of overall time.

The broadcast time and MTBF are properties of the underlying system, while the election timeout is something we must choose. Raft’s RPCs typically require the recipient to persist information to stable storage, so the broadcast time may range from 0.5–20 ms, depending on storage technology. As a result, the election timeout is likely to be somewhere between 10–500 ms. Typical server MTBFs are several months or more, which easily satisfies the timing requirement. Chapter 9 explores how to set the election timeout and its impact on availability and leader election performance in more detail.

### 3.10 Leadership transfer extension

This section describes an optional extension to Raft that allows one server to transfer its leadership to another. Leadership transfer could be useful in two types of situations:

1. Sometimes the leader must step down. For example, it may need to reboot for maintenance, or it may be removed from the cluster (see Chapter 4). When it steps down, the cluster will be idle for an election timeout until another server times out and wins an election. This brief unavailability can be avoided by having the leader transfer its leadership to another server before it steps down.

2. In some cases, one or more servers may be more suitable to lead the cluster than others. For example, a server with high load would not make a good leader, or in a WAN deployment, servers in a primary datacenter may be preferred in order to minimize the latency between clients and the leader. Other consensus algorithms may be able to accommodate these preferences during leader election, but Raft needs a server with a sufficiently up-to-date log to
become leader, which might not be the most preferred one. Instead, a leader in Raft can periodically check to see whether one of its available followers would be more suitable, and if so, transfer its leadership to that server. (If only human leaders were so graceful.)

To transfer leadership in Raft, the prior leader sends its log entries to the target server, then the target server runs an election without waiting for an election timeout to elapse. The prior leader thus ensures that the target server has all committed entries at the start of its term, and, as in normal elections, the majority voting guarantees the safety properties (such as the Leader Completeness Property) are maintained. The following steps describe the process in more detail:

1. The prior leader stops accepting new client requests.

2. The prior leader fully updates the target server’s log to match its own, using the normal log replication mechanism described in Section 3.5.

3. The prior leader sends a TimeoutNow request to the target server. This request has the same effect as the target server’s election timer firing: the target server starts a new election (incrementing its term and becoming a candidate).

Once the target server receives the TimeoutNow request, it is highly likely to start an election before any other server and become leader in the next term. Its next message to the prior leader will include its new term number, causing the prior leader to step down. At this point, leadership transfer is complete.

It is also possible for the target server to fail; in this case, the cluster must resume client operations. If leadership transfer does not complete after about an election timeout, the prior leader aborts the transfer and resumes accepting client requests. If the prior leader was mistaken and the target server is actually operational, then at worst this mistake will result in an extra election, after which client operations will be restored.

This approach preserves safety by operating within the normal transitions of a Raft cluster. For example, Raft already guarantees safety even when clocks run at arbitrary speeds; when the target server receives a TimeoutNow request, it is equivalent to the target server’s clock jumping forwards quickly, which is safe. However, we have not currently implemented or evaluated this leadership transfer approach.
3.11 Conclusion

This chapter addressed all the core problems for a consensus-based system. Raft goes beyond reaching consensus on a single value, as in single-decree Paxos; it achieves consensus on a growing log of commands, which is needed to build a replicated state machine. It also includes disseminates information once agreement has been reached, so that other servers learn the log entries that have been committed. Raft achieves consensus in a practical and efficient way by electing a cluster leader to unilaterally make decisions and transmitting only the necessary log entries when a new leader comes to power. We have implemented the ideas of Raft in LogCabin, a replicated state machine (described in Chapter 10).

Raft uses only a small amount of mechanism to address the full consensus problem. For example, it uses only two RPCs (RequestVote and AppendEntries). Perhaps surprisingly, creating a compact algorithm/implementation was not an explicit goal for Raft. Rather, it is a result of our design for understandability, where every bit of mechanism must be fully motivated and explained. We found that redundant or meandering mechanism is hard to motivate, so it naturally gets purged in the design process.

Unless we felt confident that a particular problem would affect a large fraction of Raft deployments, we did not address it in Raft. As a result, parts of Raft may appear naïve. For example, servers in Raft detect a split vote by waiting for an election timeout; in principle, they could often detect and even resolve split votes sooner by counting the votes granted to any candidate. We chose not to develop this optimization for Raft, since it adds complexity but probably brings no practical benefit: split votes are rare in a well-configured deployment. Other parts of Raft may appear overly conservative. For example, a leader only directly commits an entry from its current term, even though in some special cases it could safely commit entries from prior terms. Applying a more complex commitment rule would harm understandability and would not have a significant effect on performance; commitment is only delayed briefly with the current rule. In discussing Raft with others, we found that many people cannot help but think of such optimizations and propose them, but when the goal is understandability, premature optimizations should be left out.

Inevitably, this chapter might have left out some features or optimizations that turn out to be useful in practice. As implementers gain more experience with Raft, they will learn when and why certain additional features may be useful, and they may need to implement these for some practical deployments. Throughout the chapter, we sketched a few optional extensions that we currently think are unnecessary but that may help guide implementers should the need arise. By focusing
on understandability, we hope to have provided a solid foundation for implementers to adjust Raft according to their experiences. Since Raft works in our testing environment, we expect these to be straightforward extensions rather than fundamental changes.
Chapter 4

Cluster membership changes

Up until now we have assumed that the cluster configuration (the set of servers participating in the consensus algorithm) is fixed. In practice, it will occasionally be necessary to change the configuration, for example to replace servers when they fail or to change the degree of replication. This could be done manually, using one of two approaches:

- Configuration changes could be done by taking the entire cluster off-line, updating configuration files, and then restarting the cluster. However, this would leave the cluster unavailable during the changeover.

- Alternatively, a new server could replace a cluster member by acquiring its network address. However, the administrator must guarantee that the replaced server will never come back up, or else the system would lose its safety properties (for example, there would be an extra vote).

Both of these approaches to membership changes have significant downsides, and if there are any manual steps, they risk operator error.

In order to avoid these issues, we decided to automate configuration changes and incorporate them into the Raft consensus algorithm. Raft allows the cluster to continue operating normally during changes, and membership changes can be implemented with only a few extensions to the basic consensus algorithm. Figure 4.1 summarizes the RPCs used to change cluster membership, whose elements are described in the remainder of this chapter.
Preserving safety is the first challenge for configuration changes. For the mechanism to be safe, there must be no point during the transition where it is possible for two leaders to be elected for the same term. If a single configuration change adds or removes many servers, switching the cluster directly from the old configuration to the new configuration can be unsafe; it isn’t possible to atomically switch all of the servers at once, so the cluster can potentially split into two independent majorities during the transition (see Figure 4.2).

Most membership change algorithms introduce additional mechanism to deal with such problems. This is what we did for Raft initially, but we later discovered a simpler approach, which is to disallow membership changes that could result in disjoint majorities. Thus, Raft restricts the types of changes that are allowed: only one server can be added or removed from the cluster at a time. More complex changes in membership are implemented as a series of single-server changes. Most of this chapter describes the single-server approach, which is easier to understand than our original approach. For completeness, Section 4.3 describes the original approach, which incurs additional complexity to handle arbitrary configuration changes. We implemented the more complex approach in LogCabin prior to discovering the simpler single-server change approach; it still uses the more
Figure 4.2: Switching directly from one configuration to another can be unsafe because different servers will switch at different times. In this example, the cluster grows from three servers to five. Unfortunately, there is a point in time where two different leaders can be elected for the same term, one with a majority of the old configuration \( C_{\text{old}} \) and another with a majority of the new configuration \( C_{\text{new}} \).

(a) Adding one server to a 4-server cluster. (b) Adding one server to a 3-server cluster.

(c) Removing one server from a 5-server cluster. (d) Removing one server from a 4-server cluster.

Figure 4.3: The addition and removal of a single server from an even- and an odd-sized cluster. In each figure, the blue rectangle shows a majority of the old cluster, and the red rectangle shows a majority of the new cluster. In every single-server membership change, an overlap between any majority of the old cluster and any majority of the new cluster is preserved, as needed for safety. For example in (b), a majority of the old cluster must include two of the left three servers, and a majority of the new cluster must include three of the servers in the new cluster, of which at least two must come from the old cluster.
complex approach at the time of this writing.

When adding a single server to a cluster or removing a single server from a cluster, any majority of the old cluster overlaps with any majority of the new cluster; see Figure 4.3. This overlap prevents the cluster from splitting into two independent majorities; in terms of the safety argument of Section 3.6.3, it guarantees the existence of “the voter”. Thus, when adding or removing just a single server, it is safe to switch directly to the new configuration. Raft exploits this property to change cluster membership safely using little additional mechanism.

Cluster configurations are stored and communicated using special entries in the replicated log. This leverages the existing mechanisms in Raft to replicate and persist configuration information. It also allows the cluster to continue to service client requests while configuration changes are in progress, by imposing ordering between configuration changes and client requests (while allowing both to be replicated concurrently in a pipeline and/or in batches).

When the leader receives a request to add or remove a server from its current configuration ($C_{\text{old}}$), it appends the new configuration ($C_{\text{new}}$) as an entry in its log and replicates that entry using the normal Raft mechanism. The new configuration takes effect on each server as soon as it is added to that server’s log: the $C_{\text{new}}$ entry is replicated to the $C_{\text{new}}$ servers, and a majority of the new configuration is used to determine the $C_{\text{new}}$ entry’s commitment. This means that servers do not wait for configuration entries to be committed, and each server always uses the latest configuration found in its log.

The configuration change is complete once the $C_{\text{new}}$ entry is committed. At this point, the leader knows that a majority of the $C_{\text{new}}$ servers have adopted $C_{\text{new}}$. It also knows that any servers that have not moved to $C_{\text{new}}$ can no longer form a majority of the cluster, and servers without $C_{\text{new}}$ cannot be elected leader. Commitment of $C_{\text{new}}$ allows three things to continue:

1. The leader can acknowledge the successful completion of the configuration change.

2. If the configuration change removed a server, that server can be shut down.

3. Further configuration changes can be started. Before this point, overlapped configuration changes could degrade to unsafe situations like the one in Figure 4.2.

As stated above, servers always use the latest configuration in their logs, regardless of whether that configuration entry has been committed. This allows leaders to easily avoid overlapping configuration changes (the third item above), by not beginning a new change until the previous change’s entry has committed. It is only safe to start another membership change once a majority of the old
cluster has moved to operating under the rules of $C_{\text{new}}$. If servers adopted $C_{\text{new}}$ only when they learned that $C_{\text{new}}$ was committed, Raft leaders would have a difficult time knowing when a majority of the old cluster had adopted it. They would need to track which servers know of the entry’s commitment, and the servers would need to persist their commit index to disk; neither of these mechanisms is required in Raft. Instead, each server adopts $C_{\text{new}}$ as soon as that entry exists in its log, and the leader knows it’s safe to allow further configuration changes as soon as the $C_{\text{new}}$ entry has been committed. Unfortunately, this decision does imply that a log entry for a configuration change can be removed (if leadership changes); in this case, a server must be prepared to fall back to the previous configuration in its log.

In Raft, it is the caller’s configuration that is used in reaching consensus, both for voting and for log replication:

- A server accepts AppendEntries requests from a leader that is not part of the server’s latest configuration. Otherwise, a new server could never be added to the cluster (it would never accept any log entries preceding the configuration entry that adds the server).

- A server also grants its vote to a candidate that is not part of the server’s latest configuration (if the candidate has a sufficiently up-to-date log and a current term). This vote may occasionally be needed to keep the cluster available. For example, consider adding a fourth server to a three-server cluster. If one server were to fail, the new server’s vote would be needed to form a majority and elect a leader.

Thus, servers process incoming RPC requests without consulting their current configurations.

### 4.2 Availability

Cluster membership changes introduce several issues in preserving the cluster’s availability. Section 4.2.1 discusses catching up new servers before they’re added to the cluster, so that they do not stall commitment of new log entries; Section 4.2.2 addresses how to phase out an existing leader if it is removed from the cluster; and Section 4.2.3 describes how to prevent removed servers from disrupting the leader of the new cluster. Finally, Section 4.2.4 closes with an argument for why the resulting membership change algorithm is sufficient to preserve availability during any membership change.
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Figure 4.4: Examples of how adding servers with empty logs can put availability at risk. The figures show the servers’ logs in two different clusters. Each cluster starts out with three servers, S1–S3. In (a), S4 is added, then S3 fails. The cluster should be able to operate normally after one failure, but it loses availability: it needs three of the four servers to commit a new entry, but S3 has failed and S4’s log is too far behind to append new entries. In (b), S4–S6 are added in quick succession. Committing the configuration entry that adds S6 (the third new server) requires four servers’ logs to store that entry, but S4–S6 have logs that are far behind. Neither cluster will be available until the new servers’ logs are caught up.

4.2.1 Catching up new servers

When a server is added to the cluster, it typically will not store any log entries. If it is added to the cluster in this state, its log could take quite a while to catch up to the leader’s, and during this time, the cluster is more vulnerable to unavailability. For example, a three-server cluster can normally tolerate one failure with no loss in availability. However, if a fourth server with an empty log is added to the same cluster and one of the original three servers fails, the cluster will be temporarily unable to commit new entries (see Figure 4.4(a)). Another availability issue can occur if many new servers are added to a cluster in quick succession, where the new servers are needed to form a majority of the cluster (see Figure 4.4(b)). In both cases, until the new servers’ logs were caught up to the leader’s, the clusters would be unavailable.

In order to avoid availability gaps, Raft introduces an additional phase before the configuration change, in which a new server joins the cluster as a non-voting member. The leader replicates log entries to it, but it is not yet counted towards majorities for voting or commitment purposes. Once the new server has caught up with the rest of the cluster, the reconfiguration can proceed as described above. (The mechanism to support non-voting servers can also be useful in other contexts;
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(a) Start of round 2.

(b) End of round 2.

Figure 4.5: To catch up a new server, the replication of entries to the new server is split into rounds. Each round completes once the new server has all of the entries that the leader had in its log at the start of the round. By then, however, the leader may have received new entries; these are replicated in the next round.

for example, it can be used to replicate the state to a large number of servers, which can serve read-only requests with relaxed consistency.)

The leader needs to determine when a new server is sufficiently caught up to continue with the configuration change. This requires some care to preserve availability: if the server is added too soon, the cluster’s availability may be at risk, as described above. Our goal was to keep any temporary unavailability below an election timeout, since clients must already be able to tolerate occasional unavailability periods of that magnitude (in case of leader failures). Moreover, if possible, we wanted to minimize unavailability further by bringing the new server’s log even closer to the leader’s.

The leader should also abort the change if the new server is unavailable or is so slow that it will never catch up. This check is important: Lamport’s ancient Paxon government broke down because they did not include it. They accidentally changed the membership to consist of only drowned sailors and could make no more progress [48]. Attempting to add a server that is unavailable or slow is often a mistake. In fact, our very first configuration change request included a typo in a network port number; the system correctly aborted the change and returned an error.

We suggest the following algorithm to determine when a new server is sufficiently caught up to add to the cluster. The replication of entries to the new server is split into rounds, as shown in Figure 4.5. Each round replicates all the log entries present in the leader’s log at the start of the round to the new server’s log. While it is replicating entries for its current round, new entries may arrive at the leader; it will replicate these during the next round. As progress is made, the round durations shrink in time. The algorithm waits a fixed number of rounds (such as 10). If the last round lasts less than an election timeout, then the leader adds the new server to the cluster, under the assumption that there are not enough unreplicated entries to create a significant availability gap.
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Otherwise, the leader aborts the configuration change with an error. The caller may always try again (it will be more likely to succeed the next time, since the new server’s log will already be partially caught up).

As the first step to catching up a new server, the leader must discover that the new server’s log is empty. With a new server, the consistency check in AppendEntries will fail repeatedly until the leader’s nextIndex finally drops to one. This back-and-forth might be the dominant factor in the performance of adding a new server to the cluster (after this phase, log entries can be transmitted to the follower with fewer RPCs by using batching). Various approaches can make nextIndex converge to its correct value more quickly, including those described in Chapter 3. The simplest approach to solving this particular problem of adding a new server, however, is to have followers return the length of their logs in the AppendEntries response; this allows the leader to cap the follower’s nextIndex accordingly.

4.2.2 Removing the current leader

If the existing leader is asked to remove itself from the cluster, it must step down at some point. One straightforward approach is to use the leadership transfer extension described in Chapter 3: a leader that is asked to remove itself would transfer its leadership to another server, which would then carry out the membership change normally.

We initially developed a different approach for Raft, in which the existing leader carries out the membership change to remove itself, then it steps down. This puts Raft in a somewhat awkward mode of operation while the leader temporarily manages a configuration in which it is not a member. We initially needed this approach for arbitrary configuration changes (see Section 4.3), where the old configuration and the new configuration might not have any servers in common to which leadership could be transferred. The same approach is also viable for systems that do not implement leadership transfer.

In this approach, a leader that is removed from the configuration steps down once the $C_{\text{new}}$ entry is committed. If the leader stepped down before this point, it might still time out and become leader again, delaying progress. In an extreme case of removing the leader from a two-server cluster, the server might even have to become leader again for the cluster to make progress; see Figure 4.6. Thus, the leader waits until $C_{\text{new}}$ is committed to step down. This is the first point when the new configuration can definitely operate without participation from the removed leader: it will always be possible for the members of $C_{\text{new}}$ to elect a new leader from among themselves. After the removed leader steps down, a server in $C_{\text{new}}$ will time out and win an election. This small availability gap
Figure 4.6: Until the $C_{\text{new}}$ entry has been committed, a removed server may need to lead the cluster to make progress. The figure shows the removal of S1 from a two-server cluster. S1 is currently leader. S1 should not step down quite yet; it is still needed as leader. S2 cannot become leader until it receives the $C_{\text{new}}$ entry from S1 (since S2 still needs S1’s vote to form a majority of $C_{\text{old}}$, and S1 won’t grant its vote to S2 because S2’s log is less up-to-date). Should be tolerable, since similar availability gaps arise when leaders fail.

This approach leads to two implications about decision-making that are not particularly harmful but may be surprising. First, there will be a period of time (while it is committing $C_{\text{new}}$) when a leader can manage a cluster that does not include itself; it replicates log entries but does not count itself in majorities. Second, a server that is not part of its own latest configuration should still start new elections, as it might still be needed until the $C_{\text{new}}$ entry is committed (as in Figure 4.6). It does not count its own vote in elections unless it is part of its latest configuration.

4.2.3 Disruptive servers

Without additional mechanism, servers not in $C_{\text{new}}$ can disrupt the cluster. Once the cluster leader has created the $C_{\text{new}}$ entry, a server that is not in $C_{\text{new}}$ will no longer receive heartbeats, so it will time out and start new elections. Furthermore, it will not receive the $C_{\text{new}}$ entry or learn of that entry’s commitment, so it will not know that it has been removed from the cluster. The server will send RequestVote RPCs with new term numbers, and this will cause the current leader to revert to follower state. A new leader from $C_{\text{new}}$ will eventually be elected, but the disruptive server will time out again and the process will repeat, resulting in poor availability. If multiple servers have been removed from the cluster, the situation could degrade further.

Our first idea for eliminating disruptions was that, if a server is going to start an election, it would first check that it wouldn’t be wasting everyone’s time—that it had a chance to win the election. This introduced a new phase to elections, called the Pre-Vote phase. A candidate would first ask other servers whether its log was up-to-date enough to get their vote. Only if the candidate
Figure 4.7: An example of how a server can be disruptive even before the \( C_{\text{new}} \) log entry has been committed, and the Pre-Vote phase doesn’t help. The figure shows the removal of S1 from a four-server cluster. S4 is leader of the new cluster and has created the \( C_{\text{new}} \) entry in its log, but it hasn’t yet replicated that entry. Servers in the old cluster no longer receive heartbeats from S4. Even before \( C_{\text{new}} \) is committed, S1 can time out, increment its term, and send this larger term number to the new cluster, forcing S4 to step down. The Pre-Vote algorithm does not help, since S1’s log is as up-to-date as a majority of either cluster.

believed it could get votes from a majority of the cluster would it increment its term and start a normal election.

Unfortunately, the Pre-Vote phase does not solve the problem of disruptive servers: there are situations where the disruptive server’s log is sufficiently up-to-date, but starting an election would still be disruptive. Perhaps surprisingly, these can happen even before the configuration change completes. For example, Figure 4.7 shows a server that is being removed from a cluster. Once the leader creates the \( C_{\text{new}} \) log entry, the server being removed could be disruptive. The Pre-Vote check does not help in this case, since the server being removed has a log that is more up-to-date than a majority of either cluster. (Though the Pre-Vote phase does not solve the problem of disruptive servers, it does turn out to be a useful idea for improving the robustness of leader election in general; see Chapter 9.)

Because of this scenario, we now believe that no solution based on comparing logs alone (such as the Pre-Vote check) will be sufficient to tell if an election will be disruptive. We cannot require a server to check the logs of every server in \( C_{\text{new}} \) before starting an election, since Raft must always be able to tolerate faults. We also did not want to assume that a leader will reliably replicate entries fast enough to move past the scenario shown in Figure 4.7 quickly; that might have worked in practice, but it depends on stronger assumptions that we prefer to avoid about the performance of finding
where logs diverge and the performance of replicating log entries.

Raft’s solution uses heartbeats to determine when a valid leader exists. In Raft, a leader is considered active if it is able to maintain heartbeats to its followers (otherwise, another server will start an election). Thus, servers should not be able to disrupt a leader whose cluster is receiving heartbeats. We modify the RequestVote RPC to achieve this: if a server receives a RequestVote request within the minimum election timeout of hearing from a current leader, it does not update its term or grant its vote. It can either drop the request, reply with a vote denial, or delay the request; the result is essentially the same. This does not affect normal elections, where each server waits at least a minimum election timeout before starting an election. However, it helps avoid disruptions from servers not in \( C_{\text{new}} \): while a leader is able to get heartbeats to its cluster, it will not be deposed by larger term numbers.

This change conflicts with the leadership transfer mechanism as described in Chapter 3, in which a server legitimately starts an election without waiting an election timeout. In that case, RequestVote messages should be processed by other servers even when they believe a current cluster leader exists. Those RequestVote requests can include a special flag to indicate this behavior (“I have permission to disrupt the leader—it told me to!”).

### 4.2.4 Availability argument

This section argues that the above solutions are sufficient to maintain availability during membership changes. Since Raft’s membership changes are leader-based, we show that the algorithm will be able to maintain and replace leaders during membership changes and that the leader(s) will both service client requests and complete the configuration changes. We assume, among other things, that a majority of the old configuration is available (at least until \( C_{\text{new}} \) is committed) and that a majority of the new configuration is available.

1. A leader can be elected at all steps of the configuration change:

   - If the available server with the most up-to-date log in the new cluster has the \( C_{\text{new}} \) entry, it can collect votes from a majority of \( C_{\text{new}} \) and become leader.
   - Otherwise, the \( C_{\text{new}} \) entry must not yet be committed. The available server with the most up-to-date log among both the old and new clusters can collect votes from a majority of \( C_{\text{old}} \) and a majority of \( C_{\text{new}} \), so no matter which configuration it uses, it can become leader.
2. A leader is maintained once elected, assuming its heartbeats get through to its configuration, unless it intentionally steps down because it is not in $C_{\text{new}}$ but has committed $C_{\text{new}}$.

- If a leader can reliably send heartbeats to its own configuration, then neither it nor its followers will adopt a higher term: they will not time out to start any new elections, and they will ignore any RequestVote messages with a higher term from other servers. Thus, the leader will not be forced to step down.

- If a server that is not in $C_{\text{new}}$ commits the $C_{\text{new}}$ entry and steps down, Raft will then elect a new leader. It is likely that this new leader will be part of $C_{\text{new}}$, allowing the configuration change to complete. However, there is some (small) risk that the server that stepped down might become leader again. If it was elected again, it would confirm the commitment of the $C_{\text{new}}$ entry and soon step down, and it is again likely that a server in $C_{\text{new}}$ would succeed the next time.

3. The leader(s) will service client requests throughout the configuration change.

- Leaders can continue to append client requests to their logs throughout the change.

- Since new servers are caught up before being added to the cluster, a leader can advance its commit index and reply to clients in a timely manner.

4. The leader(s) will progress towards and complete the configuration change by committing $C_{\text{new}}$, and, if necessary, stepping down to allow a server in $C_{\text{new}}$ to become leader.

Therefore, under the above assumptions, the mechanisms described in this section are sufficient to preserve availability during any membership change.

4.3 Arbitrary configuration changes using joint consensus

This section presents a more complex approach to cluster membership changes that handles arbitrary changes to the configuration at one time. For example, two servers can be added to a cluster at once, or all of the servers in a five-server cluster can be replaced at once. This was the first approach to membership changes that we came up with, and it is described only for completeness. Now that we know about the simpler single-server approach, we recommend that one instead, since handling arbitrary changes requires extra complexity. Arbitrary changes are typically the way membership changes are assumed to operate in the literature, but we don’t think this flexibility is needed in real
systems, where a series of single-server changes can change the cluster membership to any desired configuration.

To ensure safety across arbitrary configuration changes, the cluster first switches to a transitional configuration we call joint consensus; once the joint consensus has been committed, the system then transitions to the new configuration. The joint consensus combines both the old and new configurations:

- Log entries are replicated to all servers in both configurations.
- Any server from either configuration may serve as leader.
- Agreement (for elections and entry commitment) requires separate majorities from both the old and new configurations. For example, when changing from a cluster of 3 servers to a different cluster of 9 servers, agreement requires both 2 of the 3 servers in the old configuration and 5 of the 9 servers in the new configuration.

The joint consensus allows individual servers to transition between configurations at different times without compromising safety. Furthermore, joint consensus allows the cluster to continue servicing client requests throughout the configuration change.

This approach extends the single-server membership change algorithm with an intermediate log entry for the joint configuration; Figure 4.8 illustrates the process. When the leader receives a
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request to change the configuration from $C_{\text{old}}$ to $C_{\text{new}}$, it stores the configuration for joint consensus ($C_{\text{old,new}}$ in the figure) as a log entry and replicates that entry using the normal Raft mechanism. As with the single-server configuration change algorithm, each server starts using a new configuration as soon as it stores the configuration in its log. This means that the leader will use the rules of $C_{\text{old,new}}$ to determine when the log entry for $C_{\text{old,new}}$ is committed. If the leader crashes, a new leader may be chosen under either $C_{\text{old}}$ or $C_{\text{old,new}}$, depending on whether the winning candidate has received $C_{\text{old,new}}$. In any case, $C_{\text{new}}$ cannot make unilateral decisions during this period.

Once $C_{\text{old,new}}$ has been committed, neither $C_{\text{old}}$ nor $C_{\text{new}}$ can make decisions without approval of the other, and the Leader Completeness Property ensures that only servers with the $C_{\text{old,new}}$ log entry can be elected as leader. It is now safe for the leader to create a log entry describing $C_{\text{new}}$ and replicate it to the cluster. Again, this configuration will take effect on each server as soon as it is seen. When the $C_{\text{new}}$ log entry has been committed under the rules of $C_{\text{new}}$, the old configuration is irrelevant and servers not in the new configuration can be shut down. As shown in Figure 4.8, there is no time when $C_{\text{old}}$ and $C_{\text{new}}$ can both make unilateral decisions; this guarantees safety.

The joint consensus approach could be generalized to allow a configuration change to begin while a prior change was still in progress. However, there would not be much practical advantage to doing this. Instead, a leader rejects additional configuration changes when a configuration change is already in progress (when its latest configuration is not committed or is not a simple majority). Changes that are rejected in this way can simply wait and try again later.

This joint consensus approach is more complex than the single-server changes precisely because it requires transitioning to and from an intermediate configuration. Joint configurations also require changes to how all voting and commitment decisions are made; instead of simply counting servers, the leader must check if the servers form a majority of the old cluster and also form a majority of the new cluster. Implementing this required finding and changing about six comparisons in our Raft implementation [86].

4.4 System integration

Raft implementations may expose the cluster membership change mechanism described in this chapter in different ways. For example, the AddServer and RemoveServer RPCs in Figure 4.1 can be invoked by administrators directly, or they can be invoked by a script that uses a series of single-server steps to change the configuration in arbitrary ways.

It may be desirable to invoke membership changes automatically in response to events like
server failures. However, this should only be done according to a reasonable policy. For example, it can be dangerous for the cluster to automatically remove failed servers, as it could then be left with too few replicas to satisfy the intended durability and fault-tolerance requirements. One reasonable approach is to have the system administrator configure a desired cluster size, and within that constraint, available servers could automatically replace failed servers.

When making cluster membership changes that require multiple single-server steps, it is preferable to add servers before removing servers. For example, to replace a server in a three-server cluster, adding one server and then removing the other allows the system to handle one server failure at all times throughout the process. However, if one server was first removed before the other was added, the system would temporarily not be able to mask any failures (since two-server clusters require both servers to be available).

Membership changes motivate a different approach to bootstrapping a cluster. Without dynamic membership, each server simply has a static file listing the configuration. With dynamic membership changes, the static configuration file is no longer needed, since the system manages configurations in the Raft log; it is also potentially error-prone (e.g., with which configuration should a new server be initialized?). Instead, we recommend that the very first time a cluster is created, one server is initialized with a configuration entry as the first entry in its log. This configuration lists only that one server; it alone forms a majority of its configuration, so it can consider this configuration committed. Other servers from then on should be initialized with empty logs; they are added to the cluster and learn of the current configuration through the membership change mechanism.

Membership changes also necessitate a dynamic approach for clients to find the cluster; this is discussed in Chapter 6.

4.5 Conclusion

This chapter described an extension to Raft for handling cluster membership changes automatically. This is an important part of a complete consensus-based system, since fault-tolerance requirements can change over time, and failed servers eventually need to be replaced.

The consensus algorithm must fundamentally be involved in preserving safety across configuration changes, since a new configuration affects the meaning of “majority”. This chapter presented a simple approach that adds or removes a single server at a time. These operations preserve safety simply, since at least one server overlaps any majority during the change. Multiple single-server
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changes may be composed to modify the cluster more drastically. Raft allows the cluster to con-
tinue operating normally during membership changes.

Preserving availability during configuration changes requires handling several non-trivial issues. In particular, the issue of a server not in the new configuration disrupting valid cluster leaders was surprisingly subtle; we struggled with several insufficient solutions based on log comparisons before settling on a working solution based on heartbeats.
Chapter 5

Log compaction

Raft’s log grows during normal operation as it incorporates more client requests. As it grows larger, it occupies more space and takes more time to replay. Without some way to compact the log, this will eventually cause availability problems: servers will either run out of space, or they will take too long to start. Thus, some form of log compaction is necessary for any practical system.

The general idea of log compaction is that much of the information in the log becomes obsolete over time and can be discarded. For example, an operation that sets $x$ to 2 is obsolete if a later operation sets $x$ to 3. Once log entries have been committed and applied to the state machine, the intermediate states and operations used to arrive at the current state are no longer needed, and they can be compacted away.

Unlike the core Raft algorithm and membership changes, different systems will have different needs when it comes to log compaction. There is no one-size-fits-all solution to log compaction for a couple of reasons. First, different systems may choose to trade off simplicity and performance to varying degrees. Second, the state machine must be intimately involved in log compaction, and state machines differ substantially in size and in whether they are based on disk or volatile memory.

The goal of this chapter is to discuss a variety of approaches to log compaction. In each approach, most of the responsibility of log compaction falls on the state machine, which is in charge of writing the state to disk and compacting the state. State machines can achieve this in different ways, which are described throughout the chapter and summarized in Figure 5.1:

- Snapshotting for memory-based state machines is conceptually the simplest approach. In snapshotting, the entire current system state is written to a snapshot on stable storage, then the entire log up to that point is discarded. Snapshotting is used in Chubby [11, 15] and
### Memory-Based Snapshots (§6.1)

**Apply entry:**
Mutate in-memory data structure

**Service read:**
Look up result in in-memory data structure

**Take snapshot:**
When Raft log size in bytes reaches 4x previous snapshot size:
1. Fork the state machine’s memory
   - In parent, continue processing requests
   - In child, serialize in-memory data structure to new snapshot file on disk
2. Discard previous snapshot file on disk
3. Discard Raft log up through child’s last applied index

**State to transfer to slow follower:**
Latest snapshot file (immutable)

### Disk-Based Snapshots (§6.2)

**Apply entry:**
1. Mutate on-disk data structure
2. Discard Raft log up through last applied index

**Service read:**
Look up result in on-disk data structure

**State to transfer to slow follower:**
Copy-on-write snapshot of on-disk data structure

### Log-Structured Merge Trees (§6.3)

**Apply entry:**
Add entry to in-memory tree

**Service read:**
1. Search for key in in-memory tree
2. Search all level 0 runs (any might contain the key)
3. For each level counting up from 1 in order, search the single run that might contain the key

**Create new run:**
When in-memory tree reaches 1 MB:
1. Serialize in-memory tree into new sorted level 0 run on disk
2. Reset in-memory tree
3. Discard Raft log up through last applied index

**Compact runs:**
When there are 4 runs at level 0:
1. Merge all level 0 runs with all level 1 runs, producing new non-overlapping level 1 runs split at 2 MB boundaries
2. Discard merged runs

When the total size of all runs at level L exceeds $10^6$ MB:
1. Merge one level L run (chosen round-robin) with all overlapping level L+1 runs, producing new non-overlapping level L+1 runs split at 2 MB boundaries
2. Discard merged runs

**State to transfer to slow follower:**
All runs on disk (immutable)

### Very Small Leader-Based Snapshots (§6.4)

**Apply entry:**
Mutate in-memory data structure

**Service read:**
Look up result in in-memory data structure

**Take snapshot:**
When Raft log size in bytes reaches 1 MB:
1. Stop accepting client requests
2. Wait until last applied index reaches end of log
3. Serialize data structure, append to new snapshot entry in log
4. Resume processing client requests
5. As each server learns the snapshot entry is committed, it discards its Raft log entries up to that entry

**State to transfer to slow follower:**
Raft log (no additional state)

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Figure 5.1: The figure shows how various approaches to log compaction can be used in Raft.

Details for log-structured merge trees in the figure are based on LevelDB [63], and details for log cleaning are based on RAMCloud [98]; rules for managing deletions are omitted.
ZooKeeper [38], and we have implemented snapshotting in LogCabin. Snapshotting is the approach presented in the most depth in this chapter, in Section 5.1.

- With disk-based state machines, a recent copy of the system state is maintained on disk as part of normal operation. Thus, the Raft log can be discarded as soon as the state machine reflects writes to disk, and snapshotting is used only when sending consistent disk images to other servers (Section 5.2).

- Incremental approaches to log compaction, such as log cleaning and log-structured merge trees, are presented in Section 5.3. These approaches write to disk efficiently, and they utilize resources evenly over time.

- Finally, Section 5.4 discusses an approach to log compaction that minimizes the mechanism required by storing snapshots directly in the log. Though easier to implement, this approach is only suitable for very small state machines.

LogCabin currently only implements the memory-based snapshotting approach (it embeds a memory-based state machine).

The various approaches to compaction share several core concepts. First, instead of centralizing compaction decisions on the leader, each server compacts the committed prefix of its log independently. This avoids having the leader transmit data to followers that already have the data in their logs. It also helps modularity: most of the complexity of log compaction is contained within the state machine and does not interact much with Raft itself. This helps keep overall system complexity to a minimum: the complexity of Raft adds to, rather than multiplies with, the complexity of log compaction. Alternative approaches that centralize compaction responsibilities on a leader are discussed further in Section 5.4 (and for very small state machines, a leader-based approach may be better).

Second, the basic interaction between the state machine and Raft involves transferring responsibility for a prefix of the log from Raft to the state machine. Sooner or later after applying entries, the state machine reflects those entries to disk in a way that can recover the current system state. Once it has done so, it tells Raft to discard the corresponding prefix of the log. Before Raft can give up responsibility for the log prefix, it must save some of its own state describing the log prefix. Specifically, Raft retains the index and term of the last entry it discarded; this anchors the rest of the log in place after the state machine’s state and allows the AppendEntries consistency check to continue to work (it needs the index and term for the entry preceding the first entry in the log).
Raft also retains the latest configuration from the discarded log prefix in order to support cluster membership changes.

Third, once Raft has discarded a prefix of the log, the state machine takes on two new responsibilities. If the server restarts, the state machine will need to load the state corresponding to the discarded log entries from disk before it can apply any entries from the Raft log. In addition, the state machine may need to produce a consistent image of the state so that it can be sent to a slow follower (one whose log is far behind the leader’s). It is not feasible to defer compaction until log entries have been “fully replicated” to every member in the cluster, since a minority of slow followers must not keep the cluster from being fully available, and new servers can be added to the cluster at any time. Thus, slow followers or new servers will occasionally need to receive their initial states over the network. Raft detects this when the next entry needed in AppendEntries has already been discarded in the leader’s log. In this case, the state machine must provide a consistent image of the state, which the leader then sends to the follower.

### 5.1 Snapshotting memory-based state machines

The first approach to snapshotting applies when the state machine’s data structures are kept in memory. This is a reasonable choice for state machines with datasets in the gigabytes or tens of gigabytes. It enables operations to complete quickly, since they never have to fetch data from disk; it is also easy to program, since rich data structures can be used and every operation can run to completion (without blocking for I/O).

Figure 5.2 shows the basic idea of snapshotting in Raft when the state machine is kept in memory. Each server takes snapshots independently, covering just the committed entries in its log. Most of the work in snapshotting involves serializing the state machine’s current state, and this is specific to a particular state machine implementation. For example, LogCabin’s state machine uses a tree as its primary data structure; it serializes this tree using a pre-order depth-first traversal (so that when applying the snapshot, parent nodes are created before their children). State machines must also serialize the information they keep for providing linearizability to clients (see Chapter 6).

Once the state machine completes writing a snapshot, the log can be truncated. Raft first stores the state it needs for a restart: the index and term of the last entry included in the snapshot and the latest configuration as of that index. Then it discards the prefix of its log up through that index. Any previous snapshots can also be discarded, as they are no longer useful.

As introduced above, the leader may occasionally need to send its state to slow followers and
Figure 5.2: A server replaces the committed entries in its log (indexes 1 through 5) with a new snapshot, which stores just the current state (variables $x$ and $y$ in this example). Before discarding entries 1 through 5, Raft saves the snapshot’s last included index (5) and term (3) to position the snapshot in the log preceding entry 6.

to new servers that are joining the cluster. In snapshotting, this state is just the latest snapshot, which the leader transfers using a new RPC called InstallSnapshot, as shown in Figure 5.3. When a follower receives a snapshot with this RPC, it must decide what to do with its existing log entries. Usually the snapshot will contain new information not already in the follower’s log. In this case, the follower discards its entire log; it is all superseded by the snapshot and may possibly have uncommitted entries that conflict with the snapshot. If, instead, the follower receives a snapshot that describes a prefix of its log (due to retransmission or by mistake), then log entries covered by the snapshot are deleted but entries following the snapshot are still valid and must be retained.

The remainder of this section discusses secondary issues for snapshotting memory-based state machines:

- Section 5.1.1 discusses how to produce snapshots in parallel with normal operations, to minimize their effects on clients;
- Section 5.1.2 discusses when to take a snapshot, balancing the space usage and the overhead of snapshotting; and
- Section 5.1.3 discusses the issues that arise in implementing snapshotting.

### 5.1.1 Snapshotting concurrently

Creating a snapshot can take a long time, both in serializing the state and in writing it to disk. For example, copying 10 GB of memory takes about one second on today’s servers, and serializing it will usually take much longer: even a solid state disk can only write about 500 MB in one second.
Figure 5.3: Leaders invoke the InstallSnapshot RPC to send snapshots to slow followers. Leaders resort to sending a snapshot only when they have already discarded the next log entry needed to replicate entries to the follower with AppendEntries. They split the snapshot into chunks for transmission. Among other benefits, this gives the follower a sign of life with each chunk, so it can reset its election timer. Each chunk is sent in order, which simplifies writing the file to disk.

The RPC includes the state needed for Raft to load the snapshot on a restart: the index and term of the last entry covered by the snapshot, and the latest configuration at that point.
Thus, both serializing and writing snapshots must be concurrent with normal operations to avoid availability gaps.

Fortunately, copy-on-write techniques allow new updates to be applied without impacting the snapshot being written. There are two approaches to this:

- State machines can be built with immutable (functional) data structures to support this. Because state machine commands would not modify the state in place, a snapshotting task could keep a reference to some prior state and write it consistently into a snapshot.

- Alternatively, the operating system’s copy-on-write support can be used (where the programming environment allows it). On Linux for example, in-memory state machines can use `fork` to make a copy of the server’s entire address space. Then, the child process can write out the state machine’s state and exit, all while the parent process continues servicing requests. The LogCabin implementation currently uses this approach.

Servers require additional memory for snapshotting concurrently, which should be planned for and managed. It is essential for state machines to have a streaming interface to the snapshot file, so that the snapshot does not have to be staged entirely in memory while it is created. Still, copy-on-write requires extra memory proportional to the fraction of the state machine state that is changed during the snapshotting process. Moreover, relying on the operating system for copy-on-write will typically use even more memory due to false sharing (for example, if two unrelated data items happen to be on the same page of memory, the second item will be duplicated even when only the first has changed). In the unfortunate event that memory capacity is exhausted during snapshotting, a server should stop accepting new log entries until it completes its snapshot; this would temporarily sacrifice the server’s availability (the cluster might still remain available), but at least it would allow the server to recover. It is better not to abort the snapshot and retry later, since the next attempts might also face the same problem. (LogCabin uses a streaming interface to disk, but it does not currently handle memory exhaustion gracefully.)

### 5.1.2 When to snapshot

Servers must decide when to snapshot. If a server snapshots too often, it wastes disk bandwidth and other resources; if it snapshots too infrequently, it risks exhausting its storage capacity, and it increases the time required to replay the log during restarts.

One simple strategy is to take a snapshot when the log reaches a fixed size in bytes. If this size is set to be significantly larger than the expected size of a snapshot, then the disk bandwidth
overhead for snapshotting will be small. However, this can result in needlessly large logs for small state machines.

A better approach involves comparing the snapshot’s size with the log’s size. If the snapshot will be many times smaller than the log, it is probably worthwhile to take a snapshot. However, calculating the size of a snapshot before it is taken can be difficult and burdensome, imposing a significant bookkeeping burden for the state machine, or requiring almost as much work as actually taking a snapshot to compute the size dynamically. Compressing snapshot files also results in space and bandwidth savings, but it is hard to predict how large the compressed output will be.

Fortunately, using the size of the previous snapshot rather than the size of the next one results in reasonable behavior. Servers take a snapshot once the size of the log exceeds the size of the previous snapshot times a configurable expansion factor. The expansion factor trades off disk bandwidth for space utilization. For example, an expansion factor of 4 results in about 20% of the disk’s bandwidth being used towards snapshotting (for every 1 byte of snapshot, 4 bytes of log entries will be written), and requires about 6 times the disk capacity as that needed to store a single copy of the state (the old snapshot, a log 4 times bigger than that, and the new snapshot being written).

Snapshotting still creates a burst of CPU and disk bandwidth usage that might impact client performance. This can be mitigated with additional hardware; for example, a second disk drive can be used to provide the additional disk bandwidth.

It may also be possible to schedule snapshots in a way that client requests never wait on a server that is snapshotting. In this approach, servers would coordinate so that only up to a minority of the servers in the cluster would snapshot at any one time (when possible). Because Raft only requires a majority of servers to commit log entries, the minority of snapshotting servers would normally have no adverse effect on clients. When a leader wished to snapshot, it would step down first, allowing another server to manage the cluster in the meantime. If this approach was sufficiently reliable, it could also eliminate the need to snapshot concurrently; servers could just be unavailable while they took their snapshots (though they would count against the cluster’s ability to mask failures). This is an exciting opportunity for future work because of its potential to both improve overall system performance and reduce mechanism.

5.1.3 Implementation concerns

This section reviews the major components needed for a snapshotting implementation and discusses the difficulties with implementing them:
• **Saving and loading snapshots**: Saving a snapshot involves serializing the state machine’s state and writing that data out to a file, while loading is the reverse process. We found this to be fairly straightforward, although it was somewhat tedious to serialize the various types of data objects from their native representations. A streaming interface from the state machine to a file on disk is useful to avoid buffering the entire state machine state in memory; it may also be beneficial to compress the stream and apply a checksum to it. LogCabin writes each snapshot to a temporary file first, then renames the file when writing is complete and has been flushed to disk; this ensures that no server loads a partially written snapshot on startup.

• **Transferring snapshots**: Transferring snapshots involves implementing the leader and follower sides of the InstallSnapshot RPC. This is fairly straightforward and may be able to share some code with saving snapshots to and loading snapshots from disk. The performance of this transfer is usually not very important (a follower that needs this state has not been participating in the commitment of entries, so it is probably not needed soon; on the other hand, if the cluster suffers additional failures, it may need to catch up the follower to restore availability).

• **Eliminating unsafe log accesses and discarding log entries**: We originally designed LogCabin without worrying about log compaction, so the code assumed that if entry \( i \) was present in the log, entries 1 through \( i - 1 \) would also be present. This is no longer true with log compaction; for example, when determining the term for the previous entry in the AppendEntries RPC, that entry might have been discarded. Removing these assumptions throughout the code required careful reasoning and testing. This would have been easier with help from a more powerful type system, if the compiler could enforce that every access to the log also handled the case that the index was out of bounds. Once we had made all the log accesses safe, discarding the prefix of the log was straightforward. Until this point, we could only test the saving, loading, and transferring snapshots in isolation, but when log entries can be safely discarded, these can all start to be exercised in system-wide tests.

• **Snapshotting concurrently with copy-on-write**: Snapshooting concurrently may require re-working the state machine or leveraging the operating system’s fork operation. LogCabin currently uses fork, which interacts poorly with threads and C++ destructors; getting this to work correctly presented some difficulty. However, it is a small amount of code and completely eliminates the need to modify the state machine’s data structures, so we think it was the right approach.
• **Deciding when to snapshot:** We recommend taking snapshots after applying every log entry during development, since that can help catch bugs quickly. Once the implementation is complete, a more useful policy of when to snapshot should be added (e.g., using statistics about the size of Raft log and the size of the last snapshot).

We found piecewise development and testing of snapshotting to be challenging. Most of these components must be in place before it is possible to discard log entries, but only then will many of the new code paths be exercised in system-wide tests. Thus, implementers should consider the order in which to implement and test these components carefully.

### 5.2 Snapshotting disk-based state machines

This section discusses a snapshotting approach for large state machines (on the order of tens or hundreds of gigabytes) that use disk as their primary location of record. These state machines behave differently in that they always have a copy of the state ready on disk in case of a crash. Applying each entry from the Raft log mutates the on-disk state and effectively arrives at a new snapshot. Thus, once an entry is applied, it can be discarded from the Raft log. (State machines can also buffer writes in memory in hopes of achieving better disk efficiency; once they are written to disk, the corresponding entries can be discarded from the Raft log.)

The main problem with disk-based state machines is that mutating state on disk can lead to poor performance. Without write buffering, it requires one or more random disk writes for every command applied, which can limit the system’s overall write throughput (and write buffering might not help much). Section 5.3 discusses incremental approaches to log compaction which write to disk more efficiently with large, sequential writes.

Disk-based state machines must be able to provide a consistent snapshot of the disk for the purpose of transmitting it to slow followers. Although they always have a snapshot on disk, they are continuously modifying it. Thus, they still require copy-on-write techniques to retain a consistent snapshot for a long enough period to transmit it. Fortunately, disk formats are almost always divided into logical blocks, so implementing copy-on-write in the state machine should be straightforward. Disk-based state machines can also rely on operating system support for their snapshots. For example, LVM (logical volume management) on Linux can be used to create snapshots of entire disk partitions [70], and some recent file systems allow snapshotting individual directories [19].

Copying a snapshot of a disk image can take a long time, and as modifications to the disk
accumulate, so does the extra disk usage required to retain the snapshot. Although we haven’t implemented disk-based snapshotting, we speculate that disk-based state machines could avoid most of this overhead by transmitting their disk contents with the following algorithm:

1. For each disk block, track the time it was last modified.

2. While continuing normal operation, transmit the entire disk contents to a follower block by block. During this process, no extra disk space is used on the leader. Since blocks are being modified concurrently, this is likely to result in an inconsistent disk image on the follower. As each block is transferred from the leader, note its last modification time.

3. Take a copy-on-write snapshot of the disk contents. Once this is taken, the leader has a consistent copy of its disk contents, but additional disk space is used as modifications to the disk occur due to continued client operations.

4. Retransmit only the disk blocks that were modified between when they were first transmitted in Step 2 and when the snapshot was taken in Step 3.

Hopefully, most of the blocks of the consistent snapshot will have already been transmitted by the time it is created in Step 3. If that is the case, the transfer in Step 4 will proceed quickly: the additional disk capacity used to retain the snapshot on the leader during Step 4 will be low, and the additional network bandwidth used during Step 4 to retransmit modified blocks will also be low.

### 5.3 Incremental cleaning approaches

Incremental approaches to compaction, such as log cleaning [97, 98] and log-structured merge trees [84, 17] (LSM trees), are also possible. Although they are more complex than snapshotting, incremental approaches have several desirable features:

- They operate on only a fraction of the data at once, so they spread the load of compaction evenly over time.

- They write to disk efficiently, both in normal operation and while compacting. They use large, sequential writes in both cases. Incremental approaches also selectively compact parts of the disk with the most reclaimable space, so they write less data to disk than snapshotting for memory-based state machines (which rewrites all of disk on every snapshot).
They can transfer consistent state snapshots fairly easily because they do not modify regions of disk in place.

Section 5.3.1 and Section 5.3.2 first describe the basics of log cleaning and LSM trees in general. Then, Section 5.3.3 discusses how they could be applied to Raft.

5.3.1 Basics of log cleaning

Log cleaning was introduced in the context of log-structured file systems [97] and has recently been proposed for in-memory storage systems such as RAMCloud [98]. In principle, log cleaning can be used for any type of data structure, though some would be harder to implement efficiently than others.

Log cleaning maintains the log as the place of record for the system’s state. The layout is optimized for sequential writing, and it makes read operations effectively random access. Thus, indexing structures are needed to locate data items to read.

In log cleaning, the log is split into consecutive regions called segments. Each pass of the log cleaner compacts the log using a three-step algorithm:

1. It first selects segments to clean that have accumulated a large fraction of obsolete entries.
2. It then copies the live entries (those that contribute to the current system state) from those segments to the head of the log.
3. Finally, it frees the storage space for the segments, making that space available for new segments.

To minimize the effect on normal operation, this process can be done concurrently [98].

As a result of copying the live entries forwards to the head of the log, the entries get to be out of order for replay. The entries can include additional information (e.g., version numbers) to recreate the correct ordering when the log is applied.

The policy of which segments are selected for cleaning has a big impact on performance; prior work proposes a cost-benefit policy that factors in not only the amount of space utilized by live entries but also how long those entries are likely to remain live [97, 98].

Determining whether entries are live is the state machine’s responsibility. For example, in a key-value store, a log entry to set a key to a particular value is live if the key exists and is currently set to the given value. Determining whether a log entry that deletes a key is live is more subtle: it is live
as long as any prior entries setting that key are present in the log. RAMCloud preserves deletion commands (called tombstones) as necessary [98], but another approach is to periodically write out a summary of the keys that are present in the current state, then all log entries regarding keys not listed are not live. Key-value stores are a fairly simple example; other state machines are possible, but unfortunately, determining liveness will be different for each.

5.3.2 Basics of log-structured merge trees

Log-structured merge trees (LSM trees) were first described by O’Neil [84] and were later popularized in distributed systems by BigTable [17]. They are used in systems such as Apache Cassandra [1] and HyperDex [27] and are available as libraries such as LevelDB [62] and its forks (e.g., RocksDB [96] and HyperLevelDB [39]).

LSM trees are tree-like data structures that store ordered key-value pairs. At a high level, they use disk similarly to log cleaning approaches: they write in large sequential strides and do not modify data on disk in place. However, instead of maintaining all state in the log, LSM trees reorganize the state for better random access.

A typical LSM tree keeps recently written keys in a small log on disk. When the log reaches a fixed size, it is sorted by key and written to a file called a run in sorted order. Runs are never modified in place, but a compaction process periodically merges multiple runs together, producing new runs and discarding the old ones. The merge is reminiscent of merge sort; when a key is in multiple input runs, only the latest version is kept, so the produced runs are more compact. The compaction strategy used in LevelDB is summarized in Figure 5.1; it segregates runs by age for efficiency (similar to log cleaning).

During normal operation, the state machine can operate on this data directly. To read a key, it first checks to see if that key was modified recently in its log, then checks each run. To avoid checking every run for a key on every lookup, some systems create a bloom filter for each run (a compact data structure which can say with certainty in some cases that a key does not appear in a run, though it may sometimes require searching a run even when a key is not present).

5.3.3 Log cleaning and log-structured merge trees in Raft

We have not attempted to implement log cleaning or LSM trees in Raft, but we speculate that both would work well. Applying LSM trees to Raft appears to be fairly straightforward. Because the Raft log already stores recent entries durably on disk, the LSM tree can keep recent data in a more
Cleaning the Raft log directly would lead to many holes, which would add significant complexity to Raft and its interaction with the state machine.

The state machine could instead structure its own data as a log and clean that log independently, without involving Raft.

Figure 5.4: Two possible approaches to log cleaning in Raft.

Applying log cleaning to Raft is less obvious. We first considered an approach in which the Raft log was divided into segments and cleaned (see Figure 5.4(a)). Unfortunately, cleaning would place a lot of holes in the log where segments were freed, which would require a modified approach to log replication. We think this approach could be made to work, but it adds significant complexity to Raft and its interaction with the state machine. Moreover, since only the leader can append to the Raft log, cleaning would need to be leader-based, which would waste the leader’s network bandwidth (this is discussed further in Section 5.4).

A better approach would be to handle log cleaning similarly to LSM trees: Raft would keep a contiguous log for recent changes, and the state machine would keep its own state as a log, but these logs would be logically distinct (see Figure 5.4(b)). When the Raft log grew to a fixed size, its new entries would be written as a new segment in the state machine’s log, and the corresponding prefix of the Raft log would be discarded. Segments in the state machine would be cleaned independently on each server, and the Raft log would remain entirely unaffected by this. We prefer this approach.
over cleaning the Raft log directly, since the complexity of log cleaning is encapsulated entirely in
the state machine (the interface between the state machine and Raft remains simple), and servers
can clean independently.

As described, this approach would require the state machine to write all of Raft’s log entries into
its own log (though it could do so in large batches). This additional copy could be optimized away
by directly moving a file consisting of log entries from Raft’s log and incorporating that file into
the state machine’s data structures. This could be a helpful optimization for performance-critical
systems, but unfortunately, it would more tightly couple the state machine and the Raft module,
since the state machine would need to understand the on-disk representation of the Raft log.

5.4 Alternative: leader-based approaches

The log compaction approaches presented in this chapter depart from Raft’s strong leader princi-
ple, since servers compact their logs without the knowledge of the leader. However, we think this
departure is justified. While having a leader helps avoid conflicting decisions in reaching consen-
sus, consensus has already been reached when snapshotting, so no decisions conflict. Data still only
flows from leaders to followers, but followers can now reorganize their data independently.

We also considered leader-based approaches to log compaction, but any benefits are usually
outweighed by performance considerations. It would be wasteful for the leader to compact its log,
then send the result to the followers, when they could just as well compact their own logs indepen-
dently. Sending the redundant state to each follower would waste network bandwidth and slow the
compaction process. Each follower already has the information needed to compact its own state,
and the leader’s outbound network bandwidth is usually Raft’s most precious (bottleneck) resource.
For memory-based snapshots, it is typically much cheaper for a server to produce a snapshot from
its local state than it is to send and receive one over the network. For incremental compaction ap-
proaches, this depends a bit more on the hardware configuration, but we also expect independent
compaction to be cheaper.

5.4.1 Storing snapshots in the log

One possible benefit to leader-based approaches is that, if all the system state could be stored in
the log, then new mechanisms to replicate and persist the state would not be needed. Thus, we
considered a leader-based approach to snapshotting in which the leader would create a snapshot and
store the snapshot as entries in the Raft log, as shown in Figure 5.5. The leader would then send
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Figure 5.5: A leader-based approach that stores the snapshot in chunks in the log, interleaved with client requests. The snapshotting process is started at the start entry, and it completes by the end entry. The snapshot is stored in several log entries between start and end. So that client requests can proceed in parallel with snapshotting, each entry is limited in size, and the rate at which the entries are appended to the log is limited: the next snapshot chunk is only appended to the log when the leader learns that the previous snapshot chunk has been committed. Once each server learns that the end entry is committed, it can discard the entries in its log up to the corresponding start entry. Replaying the log requires a two pass algorithm: the last complete snapshot is applied first, then the client requests after the snapshot’s start entry are applied.

this snapshot to each of its followers using the AppendEntries RPC. To reduce any disruption on normal operation, each snapshot would be split into many entries and interleaved with normal client commands in the log.

This would achieve better economy of mechanism than storing the snapshot outside the log, since servers would not need separate mechanisms to transfer snapshots or persist them (they would be replicated and persisted just like other log entries). However, in addition to wasting network bandwidth for followers that could just as easily produce their own snapshots, this has a serious problem. If a leader fails in the middle of creating a snapshot, it leaves a partial snapshot in the servers’ logs. In principle this could happen repeatedly and exhaust servers’ storage capacity with garbage accumulated from numerous failed snapshotting attempts. Thus, we don’t think this mechanism is viable in practice.

5.4.2 Leader-based approach for very small state machines

For very small state machines, storing the snapshot in the log not only becomes viable but can also be simplified significantly. If the snapshot is small enough (up to about one megabyte), it can fit comfortably in a single log entry without interrupting normal operation for too long. To compact the servers’ logs in this way, the leader would:
1. Stop accepting new client requests;
2. Wait for all entries in its log to be committed and its state machine to have applied all entries in its log;
3. Take a snapshot (synchronously);
4. Append the snapshot into a single log entry at the end of its log; and
5. Resume accepting new client requests.

Once each server learned that the snapshot entry was committed, it could discard every entry before the snapshot in its log. This approach would cause a small availability gap while client requests were stopped and the snapshot entry was transferred, but its impact would be limited for very small state machines.

This simpler approach avoids the implementation effort of persisting snapshots outside the log, transferring them using a new RPC, and snapshotting concurrently. However, successful systems tend to be used more than their original designers intended, and this approach would not work well for larger state machines.

5.5 Conclusion

This chapter discussed several approaches to log compaction in Raft, which are summarized in Figure 5.1. Different approaches are suitable for different systems, depending on the size of the state machine, the level of performance required, and the amount of complexity budgeted. Raft supports a wide variety of approaches that share a common conceptual framework:

- Each server compacts the committed prefix of its log independently.
- The basic interaction between the state machine and Raft involves transferring responsibility for a prefix of the log from Raft to the state machine. Once the state machine has applied commands to disk, it instructs Raft to discard the corresponding prefix of the log. Raft retains the index and term of the last entry it discarded, along with the latest configuration as of that index.
- Once Raft has discarded a prefix of the log, the state machine takes on two new responsibilities: loading the state on a restart and providing a consistent image to transfer to a slow follower.

Snapshotting for memory-based state machines is used successfully in several production systems, including Chubby and ZooKeeper, and we have implemented this approach in LogCabin.
Although operating on an in-memory data structure is fast for most operations, performance during the snapshotting process may be significantly impacted. Snapshotting concurrently helps to hide the resource usage, and in the future, scheduling servers across the cluster to snapshot at different times might keep snapshotting from affecting clients at all.

Disk-based state machines that mutate their state in place are conceptually simple. They still require copy-on-write for transferring a consistent disk image to other servers, but this may be a small burden with disks, which naturally split into blocks. However, random disk writes during normal operation tend to be slow, so this approach will limit the system’s write throughput.

Ultimately, incremental approaches can be the most efficient form of compaction. By operating on small pieces of the state at a time, they can limit bursts in resource usage (and they can also compact concurrently). They can also avoid writing the same data out to disk repeatedly; stable data should make its way to a region of disk that does not get compacted often. While implementing incremental compaction can be complex, this complexity can be offloaded to a library such as LevelDB. Moreover, by keeping data structures in memory and caching more of the disk in memory, the performance for client operations with incremental compaction can approach that of memory-based state machines.
Chapter 6

Client interaction

This chapter describes several issues in how clients interact with a Raft-based replicated state machine:

- Section 6.1 describes how clients find the cluster, even when its set of members can change over time;
- Section 6.2 describes how clients’ requests are routed to the cluster leader for processing;
- Section 6.3 describes how Raft provides linearizable consistency [34]; and
- Section 6.4 describes how Raft can process read-only queries more efficiently.

Figure 6.1 shows the RPCs that clients use to interact with the replicated state machine; the elements of these RPCs are discussed throughout the chapter. These issues apply to all consensus-based systems, and Raft’s solutions are similar to other systems.

6.1 Finding the cluster

When Raft is exposed as a network service, clients must locate the cluster in order to interact with the replicated state machine. For clusters with fixed membership, this is straightforward; for example, the network addresses of the servers can be stored statically in a configuration file. However, finding
Figure 6.1: Clients invoke the ClientRequest RPC to modify the replicated state; they invoke the ClientQuery RPC to query the replicated state. New clients receive their client identifier using a RegisterClient RPC, which helps identify when session information needed for linearizability has been discarded. In the figure, servers that are not leaders redirect clients to the leader, and read-only requests are serviced without relying on clocks for linearizability (the text presents alternatives). Section numbers such as §6.3 indicate where particular features are discussed.
the cluster when its set of servers can change over time (as described in Chapter 4) is a bigger challenge. There are two general approaches:

1. Clients can use network broadcast or multicast to find all cluster servers. However, this will only work in particular environments that support these features.

2. Clients can discover cluster servers via an external directory service, such as DNS, that is accessible at a well-known location. The list of servers in this external system need not be consistent, but it should be inclusive: clients should always be able to find all of the cluster servers, but including a few additional servers that are not currently members of the cluster is harmless. Thus, during cluster membership changes, the external directory of servers should be updated before the membership change to include any servers soon to be added to the cluster, then updated again after the membership change is complete to remove any servers that are no longer part of the cluster.

LogCabin clients currently use DNS to find the cluster. LogCabin does not currently update DNS records automatically before and after membership changes (this is left to administrative scripts).

### 6.2 Routing requests to the leader

Client requests in Raft are processed through the leader, so clients need a way to find the leader. When a client first starts up, it connects to a randomly chosen server. If the client’s first choice is not the leader, that server rejects the request. In this case, a very simple approach is for the client to try again with another randomly chosen server until it finds the leader. If clients choose servers randomly without replacement, this naïve approach is expected to find the leader of an $n$-server cluster after $\frac{n + 1}{2}$ attempts, which may be fast enough for small clusters.

Routing requests to the leader can also be made faster with simple optimizations. Servers usually know the address of the current cluster leader, since AppendEntries requests include the leader’s identity. When a server that is not leader receives a request from a client, it can do one of two things:

1. The first option, which we recommend and which LogCabin implements, is for the server to reject the request and return to the client the address of the leader, if known. This allows the client to reconnect to the leader directly, so future requests can proceed at full speed. It also takes very little additional code to implement, since clients already need to reconnect to a different server in the event of a leader failure.
2. Alternatively, the server can proxy the client’s request to the leader. This may be simpler in some cases. For example, if a client connects to any server for read requests (see Section 6.4), then proxying the client’s write requests would save the client from having to manage a distinct connection to the leader used only for writes.

Raft must also prevent stale leadership information from delaying client requests indefinitely. Leadership information can become stale all across the system, in leaders, followers, and clients:

- **Leaders**: A server might be in the leader state, but if it isn’t the current leader, it could be needlessly delaying client requests. For example, suppose a leader is partitioned from the rest of the cluster, but it can still communicate with a particular client. Without additional mechanism, it could delay a request from that client forever, being unable to replicate a log entry to any other servers. Meanwhile, there might be another leader of a newer term that is able to communicate with a majority of the cluster and would be able to commit the client’s request. Thus, a leader in Raft steps down if an election timeout elapses without a successful round of heartbeats to a majority of its cluster; this allows clients to retry their requests with another server.

- **Followers**: Followers keep track of the leader’s identity so that they can redirect or proxy clients. They must discard this information when starting a new election or when the term changes. Otherwise, they might needlessly delay clients (for example, it would be possible for two servers to redirect to each other, placing clients in an infinite loop).

- **Clients**: If a client loses its connection to the leader (or any particular server), it should simply retry with a random server. Insisting on being able to contact the last known leader would result in unnecessary delays if that server failed.

### 6.3 Implementing linearizable semantics

As described so far, Raft provides at-least-once semantics for clients; the replicated state machine may apply a command multiple times. For example, suppose a client submits a command to a leader and the leader appends the command to its log and commits the log entry, but then it crashes before responding to the client. Since the client receives no acknowledgment, it resubmits the command to the new leader, which in turn appends the command as a new entry in its log and also commits this new entry. Although the client intended for the command to be executed once, it is executed twice.
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Figure 6.2: An example of an incorrect results that can arise from duplicated commands. A client submits a command to a replicated state machine to acquire a lock. The client’s first command acquires the lock, but the client never receives the acknowledgment. When the client retries the request, it finds that the lock is already taken.

Commands can also be applied multiple times even without the client’s involvement if the network may duplicate the client’s requests.

This issue is not unique to Raft; it occurs in most stateful distributed systems. However, these at-least-once semantics are particularly unsuitable for a consensus-based system, where clients typically need stronger guarantees. Problems from duplicated commands can manifest in subtle ways that are difficult for clients to recover from. These problems cause either incorrect results, incorrect states, or both. Figure 6.2 shows an example of an incorrect result: a state machine is providing a lock, and a client finds it is unable to acquire the lock because its original request—for which it received no acknowledgment—has already acquired the lock. An example of an incorrect state would be an increment operation, where the client intends for a value to increment by one but it instead increments by two or more. Network-level reordering and concurrent clients can lead to even more surprising results.

Our goal in Raft is to implement linearizable semantics [34], which avoid these classes of problems. In linearizability, each operation appears to execute instantaneously, exactly once, at some point between its invocation and its response. This is a strong form of consistency that is simple for clients to reason about, and it disallows commands being processed multiple times.

To achieve linearizability in Raft, servers must filter out duplicate requests. The basic idea is that servers save the results of client operations and use them to skip executing the same request multiple times. To implement this, each client is given a unique identifier, and clients assign unique
serial numbers to every command. Each server’s state machine maintains a session for each client. The session tracks the latest serial number processed for the client, along with the associated response. If a server receives a command whose serial number has already been executed, it responds immediately without re-executing the request.

Given this filtering of duplicate requests, Raft provides linearizability. The Raft log provides a serial order in which commands are applied on every server. Commands take effect instantaneously and exactly once according to their first appearance in the Raft log, since any subsequent appearances are filtered out by the state machines as described above.

This approach also generalizes to allow concurrent requests from a single client. Instead of the client’s session tracking just the client’s latest sequence number and response, it includes a set of sequence number and response pairs. With each request, the client includes the lowest sequence number for which it has not yet received a response, and the state machine then discards all responses for lower sequence numbers.

Unfortunately, sessions cannot be kept forever, as space is limited. The servers must eventually decide to expire a client’s session, but this creates two problems: how can servers agree on when to expire a client’s session, and how can they deal with an active client whose session was unfortunately expired too soon?

Servers must agree on when to expire a client’s session; otherwise, servers’ state machines could diverge from each other. For example, suppose one server expired the session for a particular client, then re-applied many of that client’s duplicated commands; meanwhile, the other servers kept the session alive and did not apply the duplicates. The replicated state machine would become inconsistent. To avoid such problems, session expiry must be deterministic, just as normal state machine operations must be. One option is to set an upper bound on the number of sessions and remove entries using an LRU (least recently used) policy. Another option is to expire sessions based on an agreed upon time source. In LogCabin, the leader augments each command that it appends to the Raft log with its current time. Servers reach agreement on this time as part of committing the log entry; then, the state machines deterministically use this time input to expire inactive sessions. Live clients issue keep-alive requests during periods of inactivity, which are also augmented with the leader’s timestamp and committed to the Raft log, in order to maintain their sessions.

The second issue is how to deal with a client that continues to operate after its session was expired. We expect this to be an exceptional situation; there is always some risk of it, however, since there is generally no way to know when clients have exited. One option would be to allocate a new session for a client any time there is no record of it, but this would risk duplicate execution of
commands that were executed before the client’s previous session was expired. To provide stricter guarantees, servers need to distinguish a new client from a client whose session was expired. When a client first starts up, it can register itself with the cluster using the RegisterClient RPC. This allocates the new client’s session and returns the client its identifier, which the client includes with all subsequent commands. If a state machine encounters a command with no record of the session, it does not process the command and instead returns an error to the client. LogCabin currently crashes the client in this case (most clients probably wouldn’t handle session expiration errors gracefully and correctly, but systems must typically already handle clients crashing).

6.4 Processing read-only queries more efficiently

Read-only client commands only query the replicated state machine; they do not change it. Thus, it is natural to ask whether these queries can bypass the Raft log, whose purpose is to replicate changes to the servers’ state machines in the same order. Bypassing the log offers an attractive performance advantage: read-only queries are common in many applications, and the synchronous disk writes needed to append entries to the log are time-consuming.

However, without additional precautions, bypassing the log could lead to stale results for read-only queries. For example, a leader might be partitioned from the rest of the cluster, and the rest of the cluster might have elected a new leader and committed new entries to the Raft log. If the partitioned leader responded to a read-only query without consulting the other servers, it would return stale results, which are not linearizable. Linearizability requires the results of a read to reflect a state of the system sometime after the read was initiated; each read must at least return the results of the latest committed write. (A system that allowed stale reads would only provide serializability, which is a weaker form of consistency.) Problems due to stale reads have already been discovered in two third-party Raft implementations [45], so this issue deserves careful attention.

Fortunately, it is possible to bypass the Raft log for read-only queries and still preserve linearizability. To do so, the leader takes the following steps:

1. If the leader has not yet marked an entry from its current term committed, it waits until it has done so. The Leader Completeness Property guarantees that a leader has all committed entries, but at the start of its term, it may not know which those are. To find out, it needs to commit an entry from its term. Raft handles this by having each leader commit a blank no-op entry into the log at the start of its term. As soon as this no-op entry is committed, the leader’s commit index will be at least as large as any other servers’ during its term.
2. The leader saves its current commit index in a local variable `readIndex`. This will be used as a lower bound for the version of the state that the query operates against.

3. The leader needs to make sure it hasn’t been superseded by a newer leader of which it is unaware. It issues a new round of heartbeats and waits for their acknowledgments from a majority of the cluster. Once these acknowledgments are received, the leader knows that there could not have existed a leader for a greater term at the moment it sent the heartbeats. Thus, the readIndex was, at the time, the largest commit index ever seen by any server in the cluster.

4. The leader waits for its state machine to advance at least as far as the readIndex; this is current enough to satisfy linearizability.

5. Finally, the leader issues the query against its state machine and replies to the client with the results.

This approach is more efficient than committing read-only queries as new entries in the log, since it avoids synchronous disk writes. To improve efficiency further, the leader can amortize the cost of confirming its leadership: it can use a single round of heartbeats for any number of read-only queries that it has accumulated.

Followers could also help offload the processing of read-only queries. This would improve the system’s read throughput, and it would also divert load away from the leader, allowing the leader to process more read-write requests. However, these reads would also run the risk of returning stale data without additional precautions. For example, a partitioned follower might not receive any new log entries from the leader for long periods of time, or even if a follower received a heartbeat from a leader, that leader might itself be deposed and not yet know it. To serve reads safely, the follower could issue a request to the leader that just asked for a current readIndex (the leader would execute steps 1–3 above); the follower could then execute steps 4 and 5 on its own state machine for any number of accumulated read-only queries.

LogCabin implements the above algorithm on leaders, and it amortizes the cost of the heartbeats across multiple read-only queries under high load. Followers in LogCabin do not currently serve read-only requests.

### 6.4.1 Using clocks to reduce messaging for read-only queries

Up until now, the approach to read-only queries presented has provided linearizability in an asynchronous model (where clocks, processors, and messages can all operate at arbitrary speeds). This
To use clocks instead of messages for read-only queries, the leader would use the normal heartbeat mechanism to maintain a lease. Once the leader’s heartbeats were acknowledged by a majority of the cluster, it would extend its lease to $\text{start} + \frac{\text{election timeout}}{\text{clock drift bound}}$, since the followers shouldn’t time out before then. While the leader held its lease, it would service read-only queries without communication.

The lease approach assumes a bound on clock drift across servers (over a given time period, no server’s clock increases more than this bound times any other). Discovering and maintaining this bound might present operational challenges (e.g., due to scheduling and garbage collection pauses, virtual machine migrations, or clock rate adjustments for time synchronization). If the assumptions are violated, the system could return arbitrarily stale information.

Fortunately, a simple extension can improve the guarantee provided to clients, so that even under asynchronous assumptions (even if clocks were to misbehave), each client would see the replicated
state machine progress monotonically (sequential consistency). For example, a client would not see the state as of log index $n$, then change to a different server and see only the state as of log index $n - 1$. To implement this guarantee, servers would include the index corresponding to the state machine state with each reply to clients. Clients would track the latest index corresponding to results they had seen, and they would provide this information to servers on each request. If a server received a request for a client that had seen an index greater than the server’s last applied log index, it would not service the request (yet).

6.5 Conclusion

This chapter discussed several issues in how clients interact with Raft. The issues of providing linearizability and optimizing read-only queries are particularly subtle in terms of correctness. Unfortunately, when the consensus literature only addresses the communication between cluster servers, it leaves these important issues out. We think this is a mistake. A complete system must interact with clients correctly, or the level of consistency provided by the core consensus algorithm will go to waste. As we’ve already seen in real Raft-based systems, client interaction can be a major source of bugs, but we hope a better understanding of these issues can help prevent future problems.
Chapter 7

Raft user study

This is the first of four chapters that each evaluate an aspect of Raft:

- This chapter evaluates Raft’s understandability,
- Chapter 8 discusses Raft’s correctness,
- Chapter 9 evaluates Raft’s leader election algorithm, and
- Chapter 10 discusses Raft’s implementations and evaluates its performance.

We designed Raft to be understandable based on our intuitions and anecdotal evidence, but we wanted to evaluate its understandability more objectively. Although measuring understandability is inherently difficult, this was important to us for two reasons. First, without an evaluation, our central claim that Raft is easy to understand would be hard to justify. Second, one of our goals was to propose understandability as a first-class feature in computer systems, so we also carried the burden of proposing a way to evaluate it.

To evaluate Raft’s understandability, we conducted an experimental study. This study compared students’ ability to answer quiz questions about Raft and Paxos after learning each algorithm. Our participants were upper-level undergraduate and graduate students at Stanford University and the University of California, Berkeley. We recorded video lectures of Raft and Paxos and created corresponding quizzes. The Raft lecture covered the basic Raft algorithm (Chapter 3) and briefly covered the joint consensus approach to arbitrary membership changes (Section 4.3); the Paxos lecture covered enough material to create an equivalent replicated state machine, including single-decree Paxos, Multi-Paxos, cluster membership changes, and a few optimizations needed in practice (such

This study involved human subjects. It was approved under exempt status by the Stanford University IRB (Institutional Review Board) as Protocol 26663.
as leader election). The lecture videos and slides are available online [88]. The quizzes tested basic understanding of the algorithms and also required students to reason about corner cases. Each student watched one video, took the corresponding quiz, watched the second video, and took the second quiz. About half of the participants did the Paxos portion first and the other half did the Raft portion first, in order to account for both individual differences in performance and experience gained from the first portion of the study. We compared participants’ scores on the two quizzes to determine whether participants showed a better understanding of Raft than Paxos.

On average, participants scored 22.6% higher on the Raft quiz than on the Paxos quiz (out of a possible 60 points, the mean Raft score was 25.7 and the mean Paxos score was 21.0). Accounting for whether people learn Paxos or Raft first, a linear regression model predicts scores 12.5 points higher on the Raft quiz than on the Paxos quiz for students with no prior Paxos experience. Section 7.4.1 analyzes the quiz results in detail.

We also surveyed participants after their quizzes to see which algorithm they felt would be easier to implement or explain. An overwhelming majority of participants reported Raft would be easier to implement and explain (33 of 41 for each question). However, these self-reported feelings may be less reliable than participants’ quiz scores. Section 7.4.2 analyzes the survey results in detail.

Our study was unconventional for systems research, and we learned many lessons while designing and conducting it. For example, in a user study, almost all of the work must be done before seeing any results; this leaves little room for error. Two sections discuss the lessons we learned. Section 7.2 explores the numerous design decisions we considered in developing our methods and materials. Section 7.5 explores how effectively the experiment convinced others of Raft’s understandability, and whether it was worth the time and effort we put into it.

### 7.1 Study questions and hypotheses

Our primary goal in the study was to show Raft’s understandability. A developer should be able to learn the Raft algorithm well enough to produce a correct implementation, without an unnecessary burden of time and effort. Unfortunately, Raft’s understandability is difficult to measure directly. There is no established measure for understandability, and we have no way of telling whether Raft is the most understandable possible algorithm.

To arrive at an experiment, we needed to formulate metrics that we could measure and hypotheses that we could test. We first needed a proxy for measuring someone’s understandability. We chose to quiz participants and measure their quiz scores (Section 7.2.3 discusses an alternative of having
participants implement the algorithms instead). Second, we needed to draw a comparison between participants’ quiz scores on Raft and on other consensus algorithms. We chose to compare Raft to Paxos, the most popular consensus algorithm used today.

We wanted to explore the following questions in our study:

1. Is Raft easier to understand than Paxos?

   We predicted students would score higher on the Raft quiz than on the Paxos quiz.

2. Which aspects of Raft are hardest to understand?

   We were interested in this question as it could help lead to further improvements in Raft’s understandability. We thought students were most likely to struggle with commitment and membership changes in Raft. We felt these were the most complex and difficult aspects of Raft to explain, so students were most likely to have difficulty understanding them (this predates Raft’s simpler single-server membership change algorithm). We also felt that Paxos’ $\alpha$-based membership approach was simpler to explain (though the secondary issues it leaves unsolved are significant).

3. How does knowing Paxos affect learning Raft, and vice versa?

   We predicted that students would generally score higher on their second quiz. We had two reasons for this. First, consensus algorithms share fundamental concepts, and students should be able to grasp a concept more easily when seeing it a second time. Second, since the lectures and quizzes followed the same format, we thought students would gain useful experience during the first lecture and quiz.

4. Do people prefer to use Raft over alternatives?

   We predicted Raft’s understandability would result in a preference to implement and explain Raft.

### 7.2 Discussion about the methods

Because there is little precedent for this sort of experiment in computer systems literature, we reasoned through many our experimental design decisions from first principles. We are especially thankful for Scott Klemmer’s valuable help during this process. This section explains why we arrived at our methods by describing the alternatives we considered for each decision, including:
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- Our choice of participants and how to motivate their participation (Section 7.2.1),
- How to teach the algorithms to the participants (Section 7.2.2),
- How to test their understanding (Section 7.2.3),
- How to evaluate their performance (Section 7.2.4),
- What questions to ask in the survey (Section 7.2.5), and
- How to discover and fix problems in the study before starting (Section 7.2.6).

The methods we ultimately decided to use are then presented in Section 7.3 in a more formal APA (American Psychological Association) style.

One common principle we applied was to test participants at the start of the learning curve for the algorithms. We wanted to see how easily they could move from no knowledge to a moderate level of understanding. While we hoped that our participants would gain at least a basic understanding of both algorithms, we did not want to over-prepare them. Given infinite time, most participants will eventually understand any consensus algorithm. Thus, to measure a difference between algorithms, we had to test participants at the start of the learning curve. For example, this meant we faced a tension in motivating participants, as discussed in the next subsection: we wanted them to try, but we did not want them to study the algorithms extensively.

7.2.1 Participants

We invited students from both Stanford and Berkeley to participate in our study. This both increased our sample size and broadened the generality of our results. We chose to use the same materials and procedures in both schools so that we could compare participants’ performance across schools.

We considered various ways to use course grades to incentivize students to participate in the study. We wanted students to put equal effort into learning each algorithm, and we only wanted them to watch the lectures to prepare (without using outside information or studying excessively). Unfortunately, we had substantial concerns for each approach we considered to incentivize students:

- If students’ participation affected their course grades but they earned credit for even incorrect answers, we were concerned that students might not pay attention to the lectures. For example, a student who skipped the lectures but filled in the quizzes with any answer that came to mind would still receive full credit towards his/her course grade.

- If students’ scores on their quizzes affected their course grades, we were concerned that students might spend too much time preparing for the quizzes or that they might work harder on the more difficult to understand algorithm in order to earn the same grade. We wanted to
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80

Table 7.1: Study participation. The “total” column lists the number of students in each class; the “≥ one quiz” column lists the number that completed at least one quiz; the “≥ both quizzes” column lists the number that completed at least both quizzes; and the “full study” column lists the number that completed both quizzes and the survey.

<table>
<thead>
<tr>
<th>school</th>
<th>total</th>
<th>≥ one quiz</th>
<th>≥ both quizzes</th>
<th>full study</th>
<th>incentive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stanford</td>
<td>34</td>
<td>33</td>
<td>31</td>
<td>31</td>
<td>5% part. grade, final exam</td>
</tr>
<tr>
<td>Berkeley</td>
<td>46</td>
<td>16</td>
<td>12</td>
<td>11</td>
<td>none</td>
</tr>
<tr>
<td>Total</td>
<td>80</td>
<td>49</td>
<td>43</td>
<td>42</td>
<td>-</td>
</tr>
</tbody>
</table>

The total number of participants is approximate. For Berkeley, this number is based on the course email list and is likely an over-estimate (only 25 students signed up to do the homework towards the end of the course).

test participants at the start of the learning curve; we didn’t want students to understand the algorithms so well that our questions could measure no difference in understanding between the algorithms. We also wanted them to spend equal effort on each algorithm.

- If students were awarded extra course credit for participation or for good quiz scores, we were concerned that poorly performing or more stressed students might be overrepresented in our participants.

- Another idea we considered was to award students all of the course credit for scoring at least 50% on either quiz. Our concern with this approach was that it would leave too many possible explanations for quiz scores. For example, would students stop after their first algorithm if they believed they did well enough? Would students choose ahead of time to try to do well on Raft and not worry about the Paxos quiz (or vice versa)?

For the Stanford students, we ultimately decided to give full course credit (5% of the total course grade) for reasonable participation in the study. We intentionally left this definition vague, but if a student appeared to put some effort into the study, we awarded them full course credit. The students were also informed that the material might show up again on the course’s final exam. Almost every student in the Stanford class participated (see Table 7.1).

However, the only incentive for the Berkeley participants was the opportunity to learn the material. The instructors for the Berkeley class chose not to factor study participation into the course grades, and the class did not have exams. Even without additional incentives, at least one third of the students in the Berkeley class participated (see Table 7.1).
7.2.2 Teaching

We had many options in how to teach the algorithms to the participants. Not only are there many ways to teach in general, but there are also various approaches to teaching Paxos in particular. Our goal in the study was to compare the algorithms, not the ways they were conveyed. Therefore, it was important for the teaching method and style to be consistent. We wanted to convey the algorithms in similar ways, and we wanted to cover equivalent content. We also wanted participants to spend no more than a few hours per algorithm. We thought this would be reasonable to ask of our participants, and we would then be able to test them at the start of the learning curve for each algorithm.

We considered using papers to teach our participants, but this had two problems. First, we could not find a suitable Paxos paper. This paper would have had to:

- Cover a relatively understandable variant of Paxos (there is no single agreed upon Paxos algorithm, but some are easier to understand than others);
- Describe it completely enough to build a replicated state machine;
- Be accessible to students with no background in the topic, without needing to understand related work first; and
- Be of similar quality, style, and length to the Raft paper.

We could have written such a paper, but it would have taken months. The second problem is that papers take many hours to read, and we wanted the participants to be able to learn the algorithms in less time.

Instead, we decided to teach the participants through lectures. We estimated that we could cover enough material for each algorithm in a one-hour lecture. This was short enough that it didn’t unduly burden our participants, yet it was long enough to cover a significant amount of material at a comfortable pace. It was also short enough that we could still quiz participants towards the start of the learning curve for the algorithms.

We chose to have John Ousterhout give the lectures for both algorithms, rather than using a different lecturer for Paxos. In trying to maximize consistency across the algorithms, we considered the following factors in our decision:

- **Expertise:** We wanted an equivalent level of expertise on each algorithm, and we didn’t consider ourselves experts on Paxos before the study. We could have brought in an expert on Paxos to give the Paxos lecture. Instead, Ousterhout based his slides on those of experts, and in preparing for the Paxos lecture, we believe we learned Paxos well enough to consider ourselves sufficiently knowledgeable in it.
• **Teaching style and ability:** Ousterhout was able to keep this very consistent across his two lectures, whereas we might have struggled with different teaching styles and abilities if a separate lecturer gave the Paxos lecture.

• **Lecture quality:** Ousterhout giving both lectures raises concerns that he might not have put in the effort to produce an equally good Paxos lecture. However, he tried to produce equivalent lectures, and this is mitigated by basing his Paxos slides off of those of experts. (Also, the Raft lecture had known deficiencies during the study: we made some last-minute changes to fix a bug in it that could have been clearer if we had more time. Moreover, it presented the more complex form of membership changes, as it predated the simpler single-server change algorithm.) Balancing lecture quality might have been more difficult with a different Paxos lecturer, since the second lecturer may not have been as committed to the study.

We wanted to teach a variant of Paxos that was relatively understandable and complete, while staying true to the fundamentals of Paxos. Unfortunately, there is no agreed upon variant of Paxos; different instructors disagree on which variant to teach. We ultimately settled on a variant from David Mazières [78], which is not only efficient but also relatively understandable. However, we used Leslie Lamport’s $\alpha$ approach [49] to reconfiguration rather than Mazières’s. Although Lamport’s approach has the undesirable property that it limits Paxos’s concurrency during normal operation, we (including Mazières) believe its basic idea is easier to understand than Mazières’s and other approaches.

We recorded both lectures on video rather than having John Ousterhout present them in person. There were several advantages to recording them:

• We could fit more material in the same amount of time, since we could re-record segments when we made mistakes.

• We were able to debug problems with our lectures during the two pilot studies we ran for each algorithm (see Section 7.2.6). Having them recorded allowed us to catch issues and fix them reliably.

• Participants could watch the lectures in different orders and still see the same exact material.

• Students could watch the lectures at their own pace and at their own schedule. They could re-watch segments or speed up and slow down the videos as they wished. We did not enforce time limits on the lectures, so students could watch them at their own pace.

• The video lectures remain as documentation for the study and could be used in a repeated study. Other people outside the study have also used the videos to learn the algorithms on
their own (our Raft lecture has 14,480 views on YouTube as of August 2014, and our Paxos lecture has 9,200 views).

A possible disadvantage is that students could not ask questions during the lectures. On the other hand, questions would have disrupted the consistency benefits of having recorded lectures. For example, questions could lead to more material being presented in one lecture than another and could introduce additional differences between the Stanford and Berkeley groups. We also do not know how the recorded lectures affected study participation; while our Stanford participation was high, it’s possible that we could have gotten higher participation at Berkeley by scheduling the lectures in class.

We attempted to keep the video lectures fairly impersonal to reduce bias. For example, the video components only showed the lecture slides and not John Ousterhout himself. However, even Ousterhout’s voice-over may have been subtly biased (though he tried not to be). Concerned readers should review the lecture videos to decide for themselves; we do not know of any formal techniques to measure or reduce such bias.

In addition to the video lectures, we provided participants with minor additional materials for their preparation (lecture notes and algorithm summaries). We discouraged participants from learning about the algorithms on their own (for example, by reading papers), but we felt that some additional materials to review before the quiz and to reference during the quiz would be helpful for the participants. We made copies of the lecture slides available for easier reference, and we provided participants with algorithm summaries in the form of a (condensed) one-page summary for Raft and a (sparse) 3.5-page summary for Paxos. These are included in Appendix A.4.

7.2.3 Testing understanding

A key challenge of this study was how to measure participants’ understanding of the algorithms. We considered having participants implement the algorithms, which would allow us to measure their ability to build working systems more directly. If feasible, this approach would have been better than the quizzes. However, we chose not to do this because of numerous challenges. First, we estimate that implementing significant portions of Raft or Paxos would take most experts weeks. If we asked this of our participants, surely we would not have had so many, and we might not have been able to draw statistically significant conclusions. Moreover, peoples’ ability to develop systems varies greatly, so to draw statistically significant conclusions, such a study would need large sample sizes or would need each participant to implement both algorithms. Both options would be difficult
in practice because of the time commitment required of participants. Even if the participation problems were solved, it would still be challenging to measure implementations against each other. A thorough treatment would need to include metrics of correctness, code complexity, and cost, all of which are challenging to measure.

Instead, we chose to quiz participants to measure their understanding. This required less time of our participants. As a result, we were able to have each participant learn both algorithms, which made it easier to factor out individual differences in learning and test-taking abilities. Moreover, it was easy to compare participants’ performance based on their numeric quiz scores.

Our most difficult challenge in developing the quizzes was how to make them fair. We first considered using questions that applied equally to both algorithms, but such questions tended to be too obvious for one of the algorithms because it more directly covered the topic. Instead, we only used similar questions if the difficulty would also be similar.

We used the following strategy to make the quizzes fair. First, we categorized each question by difficulty:

- Easy questions were essentially recall: the answer could be found in the lecture with little or no inference. We expected students to answer nearly all of these correctly.
- Medium questions required the participant to apply an algorithm found in the lecture, but it should have been straightforward to determine which steps to apply.
- Hard questions required the participant to figure out what rules to apply, combine them in new ways, and/or extrapolate beyond the lecture material. We expected that few students would be able to answer these questions perfectly.

Questions in the same difficulty category should require about the same amount of inference and extrapolation from the lecture material. We (re-)categorized questions after the lectures were created in order to ease concerns of “teaching to the quiz”.

Second, we assigned point values to each question based on how long we expected it to take. The point values were intended to reflect how many minutes it would take a reasonably prepared student to answer the question, based on John Ousterhout’s teaching experience. For example, a question that was expected to take about five minutes was worth five points.

Third, we balanced the quizzes in categories and points. Each quiz contained 4 points of easy questions, 26 points of medium questions, and 30 points of hard questions. We also compared the questions from each quiz side-by-side to confirm that they seemed equally difficult.

We believe the quizzes we produced this way are similar in difficulty, though we have no way
to know for sure. We ask readers to decide for themselves by reviewing the quizzes found in Appendix A.

Most of the questions required open-ended short answers. We also considered using multiple choice questions, which would have been easier to grade objectively. However, we decided on the open-ended format because we feel it more effectively tests participants’ understanding, as it is less suggestive of responses.

The quizzes were limited in time so that participants were unable to become experts on the questions. We did not want to give them enough time to attempt the questions, watch the entire video again, and then revise the answers.

In order to extract the most information from the quizzes, we made them intentionally difficult. For example, we didn’t want any participants to earn a perfect score because then we wouldn’t have been able to distinguish differences between them. However, we later determined that we made the quizzes a bit too hard: the maximum score was only 46.5 out of 60. For example, most students earned 0 points on question 8 on each quiz; had we made those questions easier, we might have been able to better distinguish the differences between those students.

7.2.4 Grading

We graded the quizzes using two passes. The initial (preliminary) grading pass was more subjective, assigning grades based on perceived understanding. The second (final) pass assigned grades more objectively. The following steps summarize our procedure:

1. Diego Ongaro and John Ousterhout created a plausible rubric.
2. Ongaro graded the quizzes fairly quickly (grading all participants for a given question at a time in random order, alternating between Paxos and Raft between each question).
3. Ongaro and Ousterhout revised the rubric based on problems that arose.
4. Ongaro regraded the quizzes more carefully (grading all participants for a given question at a time in order of their preliminary scores, alternating between Paxos and Raft between each question).

The final grading rubrics are included in Appendix A along with the quiz questions.

We awarded partial credit in order to gain the most information from the quiz scores as possible. For example, a blank response demonstrates no understanding, whereas one that is on track to an answer demonstrates some understanding; we wanted to distinguish these cases. We tried to award points proportional to the understanding that the participant demonstrated in his/her answer.
There are two things we could have done better. First, we exposed ourselves to preliminary scores and results prior to completing the rubric and adjusting the grading. It would have been safer to avoid this, since it raises concerns that we might have, for example, awarded fewer points to Paxos answers during the second round of grading if we thought it was going to be a close call. Although we question this aspect of our procedure, it is not too worrisome because the grades and overall results and conclusions were essentially the same after the second pass of grading. For example, the Raft mean was 25.74 after the preliminary pass of grading and 25.72 after the final pass; the Paxos mean was 20.77 after the preliminary pass, then corrected to 20.98.

Second, we graded the quizzes ourselves, and this may have introduced bias, since we hoped that the study would show that Raft is easier to understand than Paxos. We graded ourselves because it takes expertise in the algorithms to develop rubrics and grade responses (which sometimes vary greatly from each other). Therefore, it was easiest for us to do these tasks ourselves. However, we could have hired impartial graders to confirm our grading using our rubric.

### 7.2.5 Survey

We initially considered asking participants whether they understood Raft better than Paxos, rather than quizzing them. We were informed that such a survey would not be very reliable on its own. For example, participants might respond favorably towards Raft if they believe that is our desired outcome (social desirability bias), or they may be affected by wording in the questions. Although we settled on quizzes for our primary results, there is still some value in asking participants for their opinions, so we included a short survey for participants to fill out after their second quiz.

Our survey included six questions using five-point scales (Likert items) and one open-ended question for general feedback or comments. We tried to keep the survey short to encourage participation, and the answers were easy to collect and quantify this way.

### 7.2.6 Pilots

One challenge with this type of study is that it is very costly if things go wrong and the study needs to be repeated. To mitigate this risk, we attempted to discover and iron out problems with our materials and procedures before launching the study. Thus, we conducted two pilot studies, each with two to four volunteer participants who were not part of the normal study. We included 90 points of questions on each of the pilot quizzes; that way we could try more questions than we intended to keep and would have the option to throw out bad questions (our pilot quizzes included
more easy questions than the final quizzes, but we cut most of them to shorten the quizzes). We corrected many problems with the lectures and quizzes during the pilots, and we feel that the pilot process was essential to the study’s success.

7.3 Methods

This section describes the methods of the Raft user study in a more formal APA (American Psychological Association) style. It includes many technical details that are less conceptual in nature than the topics discussed in Section 7.2 but are nevertheless important to the study.

7.3.1 Study design

The experiment employed a within-subjects design in which each participant was quizzed on both the Paxos and the Raft algorithms. To account for ordering effects, it was also counterbalanced: about half of the participants learned Paxos and took the Paxos quiz, then learned Raft and took the Raft quiz; the other half learned Raft and took the Raft quiz, then learned Paxos and took the Paxos quiz.

There were two key independent variables:
- Which algorithm (Paxos or Raft)?
- Which order (Paxos then Raft, or Raft then Paxos)?

We recorded two additional independent variables, though we hoped their effects would be minor:
- Which school did the participant come from (Stanford or Berkeley)?
- Did the participant have prior experience with Paxos?

7.3.2 Participants

We invited students from Stanford and Berkeley to participate in our study. Table 7.1 summarizes the participation from each group and how many participants completed each portion of the study.

The 33 Stanford participants were recruited from the Advanced Operating Systems (CS240) course at Stanford University offered January through March 2013 and taught by David Mazières. The students were upper-level undergraduate and graduate students, and a small number of remote professional students (SCPD). They were informed that “reasonable participation” in the study would award them 5% of their course grade. (They were also offered an alternate option should they
choose not to participate in the study.) They were also informed that questions on the material may reappear on the final exam for the course.

The 16 Berkeley participants were recruited from the Distributed Computing (CS294-91) course at the University of California, Berkeley offered January though May 2013 and taught by Ali Ghodsi. The students were mostly graduate-level (though at least one undergraduate student took the course). It was vaguely suggested to the students that they should participate in the study as part of the course, but there was no explicit incentive to encourage participation.

### 7.3.3 Materials

Participants gained access to materials for the study though a password-protected website. The website allowed participants to proceed with the study at their own pace and at any time of day. The various materials available on the website are explained in more detail next.

**Lectures**

Each algorithm had a corresponding video lecture. The lecture slides were designed and created by John Ousterhout. The Paxos lecture borrowed from slides by Lorenzo Alvisi, Ali Ghodsi, and David Mazières; the Raft lecture was based on a draft of a paper describing Raft. The slides were made available to the participants on the study website in both Microsoft PowerPoint and PDF formats. The videos used a “screencast” format: the video components showed only the slides and a stylus overlay, and the audio component consisted of Ousterhout verbally explaining the slides. Figure 7.1 shows an example of a slide with the stylus overlay. The videos were recorded in advance so that students in both ordering groups could use the same exact videos. They were made available to the students on both the YouTube video hosting website and in MP4 format for download.

The Raft lecture covered the following topics: leader election, log replication, safety, client interaction, and the joint consensus approach to membership changes. The Paxos lecture covered enough material to create an equivalent replicated state machine, including single-decree Paxos, Multi-Paxos, client interaction, membership changes, and a few optimizations needed in practice (such as leader election). Log compaction was not included in either lecture.

We aimed to create video lectures that were about one hour in length. We tried to balance the lectures so they covered the material at an equivalent level of detail with similar numbers of examples. This resulted in the Paxos lecture being slightly longer than the Raft lecture. Table 7.2 compares their lengths using various metrics.
### Figure 7.1: Example lecture slide marked up with stylus overlay. This slide comes from the Paxos lecture (it shows liveness problems that could arise if two competing proposers were too synchronized).

### Table 7.2: Various measures of length for the two lectures. The lecture word count is an approximation based on automated YouTube transcripts of the lecture videos. The percentages in parenthesis show the additional length of the Paxos lecture relative to the Raft lecture.
Supporting materials

Both lectures included optional summaries of the algorithms. For the Raft lecture, the summary was a single slide (participants needed to zoom into this slide to read it). For Paxos, a 3.5-page summary of the single-decree and Multi-Paxos algorithms was provided along with the lecture on the study website. These are included in Appendix A.4. Participants did not need to view the summaries to score well on the quizzes, but they were provided as quick reference and review materials. We did not track whether participants actually viewed the summaries.

Quizzes

Each algorithm included a web-based quiz. The quizzes and their solutions and grading rubrics are provided in Appendix A. The quizzes tested basic understanding of the algorithms and also required students to reason about corner cases. Most of the questions required open-ended short answers. Each quiz consisted of eight questions of varying difficulty (some were multi-part questions). We categorized the questions using a difficulty rating scale (see Section 7.2.3):

- The first question (4 points) was rated easy.
- The next four questions (26 points) were rated medium.
- The last three questions (30 points) were rated hard.

The point values were intended to reflect how many minutes it would take a reasonably prepared student to answer the question. Participants were given the point values, but questions on the quizzes were not explicitly labeled with their difficulty ratings.

Unfortunately, the Paxos quiz used in the study had one typo in Question 4. The original question used in the study and the correction can be found in Appendix A, along with a description of how the question was graded.

Participants were instructed to complete each quiz within 60 minutes. The website included a decreasing counter with minute-level granularity (we were advised that finer-grained counters can cause unnecessary anxiety). No technical measures were employed to force students to submit their answers within 60 minutes. At the end of 60 minutes, this counter would go negative. However, participants’ web browsers reported the full elapsed quiz time, and the server kept records of the time when the participant first opened a quiz and when he/she submitted each quiz. Only four participants went more than 10% over the time limit (we included those quizzes anyway in the results presented in Section 7.4.1).
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Survey

Following their second quiz, participants were asked to complete a short web-based survey, which can be found in Appendix A.3. It consisted of six questions using five-point scales (Linkert items) and one open-ended question for general feedback or comments. It included questions about their prior experience with Paxos and whether they would prefer to implement or explain one of the algorithms over the other.

7.3.4 Dependent measures

Participants’ performance on the quizzes formed the primary dependent measure for this study. Diego Ongaro graded the quizzes in random order according to a rubric. Ongaro was blind as to the participants’ schools during grading (and was and still is blind as to the participants’ identities). Participants’ preferences in the survey were also a dependent measure.

7.3.5 Procedure

Participants were randomly assigned to an ordering group (Paxos first or Raft first) over e-mail. This e-mail instructed the participants to complete the first quiz by 11:59 p.m. on a Monday and the second quiz by 11:59 p.m. on that Friday (though we accepted both early and late responses). The e-mail included a link to the study website and unique login credentials for each participant.

The study website included the materials described above. Participants could visit the website at any time. They were not timed as they watched the videos or studied the supporting materials. The website instructed them that they would have 60 minutes to complete the quiz once they opened it. The website saved the quiz responses frequently and reloaded them in case the participant reopened the quiz page. After submitting the second quiz, the website prompted the participant to fill out the survey.

7.4 Results

This section presents the results obtained from our experiment. Section 7.4.1 describes the quiz results, and Section 7.4.2 describes the survey results.
Figure 7.2: CDF of participants’ quiz scores. Each curve shows the fraction of participants who scored at most that many points (right/lower is better); for example, about 47% of participants scored up to 25 points on the Raft quiz; the remaining 53% scored higher. The maximum possible score was 60 points on each quiz. 47 participants completed the Paxos quiz; 45 completed the Raft quiz. Figure 7.7 facets this graph down by question difficulty and ordering.
7.4.1 Quizzes

Figure 7.2 shows the raw distributions of quiz scores; the Raft scores are generally greater than the Paxos scores by a few points. The mean Raft score is 4.74 points or 22.6% higher than the mean Paxos score. We used a statistical significance test to confirm this difference: we conducted an unpaired Student’s $t$-test with a one-sided hypothesis that the Raft scores were greater than the Paxos scores. This test found that the Raft scores ($M = 25.72, SD = 10.33$) were significantly greater than the Paxos scores ($M = 20.98, SD = 8.57$); $t(85.55) = 2.39, p = 0.009$. In layman’s terms, we can say with 99% confidence from our sample that the true distribution of Raft quiz scores is greater than the true distribution of Paxos quiz scores (there is only a 1% chance that we would find such a difference by random chance in identical distributions; a $p$-value less than 5% is typically considered statistically significant).

We also wanted to consider individual differences in learning and test-taking abilities. Because participants learned and took quizzes on both algorithms, we could look at each participants’ difference in quiz scores. (Six participants only took one quiz, so we exclude them here.)

Figures 7.3 and 7.4 plot individuals’ quiz scores against each other. They show that 33 of 43 of the participants scored higher on their Raft quiz than on their Paxos quiz. Figure 7.3 overlays the order in which participants learned the algorithms and took the quizzes, while Figure 7.4 overlays participants’ prior Paxos exposure. Neither of these appear to be obviously correlated with which participants scored higher on their Raft quiz.

Figure 7.5 shows the overall distribution of how participants scores differ across exams; this makes it easier to compare the overall behavior of the data. The participants’ scored a median of 6.5 points or 31.7% higher on the Raft quiz than on the Paxos quiz. We conducted a paired samples $t$-test with a one-sided hypothesis that participants Raft scores were generally greater than their Paxos scores. This test found that individuals’ Raft scores ($M = 25.73, SD = 10.56$) were significantly greater than their Paxos scores ($M = 20.79, SD = 8.64$); $t(42) = 3.39, p = 0.001$. In layman’s terms, we can say with 99.9% confidence from our sample that similar individuals will score greater on their Raft quiz than on their Paxos quiz (there is only a 0.1% chance that we would find such a difference by random chance in identical distributions).

We were also curious whether the order in which students learned the systems affected their quiz scores. Figure 7.6 shows participants’ quiz scores grouped by whether they took the Raft quiz first or second. It appears from the figure that the participants who took the Raft quiz first scored about five points higher on the Raft quiz than those who took the Paxos quiz first. To investigate this effect, we used statistical tests to determine whether the scores in the two groups truly differed for the same
Figure 7.3: A scatter plot of 43 participants’ grades comparing their performance on each quiz. Points above the diagonal (33) represent participants who scored higher on the Raft quiz. The shape and color of each point represent whether that particular participant watched the Raft lecture and took the Raft quiz first or whether he/she watched the Paxos lecture and took the Paxos quiz first. Figure 7.4 is a similar scatter plot which shows participants’ prior Paxos exposure instead.
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Figure 7.4: A scatter plot of 43 participants’ grades comparing their performance on each quiz, showing the participants’ prior Paxos exposure. The shape and color of each point represent the prior Paxos exposure that participant reported in the survey (the exact question can be found in Appendix A.3). One participant did not respond to the question (labeled “N/A”). No students reported prior Raft exposure. Figure 7.3 is a similar scatter plot which shows the order in which participants took the quizzes instead.
Figure 7.5: CDF of 43 participants’ Raft scores compared to their Paxos scores. The left graph shows participants’ relative score difference between the quizzes (an x-value of 2 means the participant’s score on the Raft quiz was twice their score on the Paxos quiz). The right graph shows the participants’ absolute score difference between the quizzes (positive values represent participants who scored higher on Raft).
Figure 7.6: Ordering effects on participants’ quiz scores.
The boxplots summarize the participants’ quiz score distributions. The top of each line is the maximum score attained, the top of each box is the 75th percentile, the middle of each box is the median, the bottom of each box is the 25th percentile, and the bottom of each line is the minimum score attained.

Dashed lines connect the quantiles on boxplots for the same quiz between different ordering groups. For example, the thick, dashed, blue line connects the median score for the Raft quiz in the group that took the Raft quiz first (left) to the median score for the Raft quiz in the group that took the Raft quiz second (right). If the ordering of the quizzes did not affect participants’ performance, these dashed lines would be nearly horizontal.

Participants’ individual quiz scores overlay each boxplot to provide further detail. Each point’s x coordinate is randomly offset to reduce overlap.
quiz: we conducted unpaired $t$-tests with two-sided hypotheses that the groups differed in either direction. These showed no statistically significant differences between the groups; such a difference for Raft could occur by random chance in 16.8% of similar experiments. However, ordering does appear to be a statistically significant factor when also considering prior Paxos experience; this is discussed next as a component of a linear regression model.

We created a linear regression model to investigate the effects of various factors on quiz scores. The model considered whether the participants were taking their first or second quiz, their prior Paxos experience, and their school. To test whether ordering and prior Paxos experience affected the Raft or Paxos quizzes differently, the linear model included two variables for each of those. Thus, the model included the following variables:

- **Quiz**: Paxos or Raft.
- **Second quiz, Raft**: the participant took the Paxos quiz before the Raft quiz, and **Quiz** is Raft.
- **Second quiz, Paxos**: the participant took the Raft quiz before the Paxos quiz, and **Quiz** is Paxos.
- **Prior Paxos experience, Raft**: the participant’s prior Paxos experience if the **Quiz** is Raft, 0 otherwise. In order to include this factor in the model, participants’ prior Paxos experience was mapped from the English answer labels found in Appendix A.3 to integers between 0 and 4.
- **Prior Paxos experience, Paxos**: the participant’s prior Paxos experience if the **Quiz** is Paxos, 0 otherwise.
- **School**: Stanford or Berkeley.

Our first model (Table 7.3) reported that the **School** factor was insignificant, so we created a second model that excludes it. The second model, shown in Table 7.4, explains 19% of the variance in quiz scores; the other 81% at least includes individual differences in learning and test-taking abilities. Accounting for ordering, this model predicts quiz scores that are 12.5 points higher on the Raft quiz than on the Paxos quiz for students with no prior Paxos experience.

The linear model also predicts higher scores on both quizzes for people who learn Raft before Paxos (6.3 points on the Raft quiz and 3.6 points on the Paxos quiz). This difference is statistically significant for the Raft quiz ($p = 0.031$) but not for the Paxos quiz ($p = 0.209$). We speculate that learning Paxos first may have confused or discouraged our participants enough that they then performed worse on the Raft quiz.

Figure 7.7 shows distributions of quiz scores broken down by question difficulty. There were only 4 points of Easy questions, so we combined those with the Medium category in the graphs.
Variable | Estimate | Std. Error | t-value | p-value
--- | --- | --- | --- | ---
Intercept | 10.61 | 3.32 | 3.19 | 0.002
Quiz is Raft | 12.99 | 4.46 | 2.92 | 0.005
Second quiz, Raft | −6.74 | 2.87 | −2.35 | 0.021
Second quiz, Paxos | 4.09 | 2.87 | 1.42 | 0.158
Prior Paxos experience, Paxos | 4.65 | 1.54 | 3.02 | 0.003
Prior Paxos experience, Raft | 3.11 | 1.54 | 2.02 | 0.046
School is Berkeley | 3.26 | 2.28 | 1.43 | 0.157

Table 7.3: Linear model of quiz grades, including school factor. This model is statistically significant ($F(6,77) = 4.48$, $p = 0.001$). It explains 20% of the variance in quiz scores (adjusted $R^2 = 0.20$).

The “Intercept” represents a constant number of points predicted as a baseline for every participant. The value of each variable is multiplied by its coefficient in the “Estimate” column; these are summed to form the predicted quiz score. For example, a Berkeley student taking her Raft quiz after having taken her Paxos quiz, with no prior Paxos experience, would be expected to receive a quiz score of $10.61 + 12.99(1) - 6.74(1) + 4.09(0) + 4.65(0) + 3.11(0) + 3.26(1) = 20.12$. The “$p$-value” represents the probability that each variable’s co-efficient does not significantly differ from 0; normally $p$-values below 0.05 are considered statistically significant. The “Std. Error” and “$t$-value” columns are used to calculate the $p$-values.

Two variables in this model are not statistically significant: “Second Quiz, Paxos” and “School is Berkeley”. In refining this model to arrive at Table 7.4, we kept the “Second Quiz, Paxos” variable because it is symmetric with the “Second Quiz, Raft” variable, which is significant. However, we dropped the “School is Berkeley” variable.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t-value</th>
<th>p-value</th>
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<td>Intercept</td>
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<td>3.31</td>
<td>3.40</td>
<td>0.001</td>
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<td>2.80</td>
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<td>2.87</td>
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<td>0.031</td>
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<tr>
<td>Second quiz, Paxos</td>
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<td>2.87</td>
<td>1.27</td>
<td>0.209</td>
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<tr>
<td>Prior Paxos experience, Paxos</td>
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<td>1.54</td>
<td>3.18</td>
<td>0.002</td>
</tr>
<tr>
<td>Prior Paxos experience, Raft</td>
<td>3.35</td>
<td>1.54</td>
<td>2.18</td>
<td>0.032</td>
</tr>
</tbody>
</table>

Table 7.4: Linear model of quiz grades, excluding school factor. This model is statistically significant ($F(5,78) = 4.90$, $p = 0.0006$). It explains 19% of the variance in quiz scores (adjusted $R^2 = 0.19$).
Figure 7.7: CDFs of 43 participants’ quiz scores, broken down by question difficulty and ordering. Each curve shows the fraction of participants who scored up to that many points (right/lower is better). The total, (all) graph is identical to Figure 7.2.

Difficulty: The easy/medium column shows the participants’ scores for the easy and medium questions on the quiz, out of a maximum possible 30 points. The hard column shows the participants’ scores for the hard questions on the quiz, out of a maximum possible 30 points. The total column shows the participants’ total scores, out of a possible 60 points. Figure 7.8 breaks this down by individual question.

Ordering: The Raft, then Paxos row shows the scores for the participants who took the Raft quiz before taking the Paxos quiz. The Paxos, then Raft row shows the scores for the participants who took the Paxos quiz before taking the Raft quiz. The (all) column shows the scores for all participants who took both quizzes.
The graphs show that almost all of the difference in scores can be attributed to questions in the Easy/Medium category, and the Hard category accounted for only a very small difference in scores. We made the hard questions too difficult: on average, participants scored only about one quarter of the possible points in the Hard category (7.45 points on average on Paxos and 7.94 points on average on Raft). Thus, we were unable to measure much difference between participants in the Hard category.

Figure 7.8 breaks the quiz scores down by individual question. Question 1 was Easy difficulty, Questions 2 through 5 were Medium difficulty, and Questions 6 through 8 were Hard difficulty, based on our categorization. There do not appear to be any individual questions in the Medium category that alone account for large differences. Although we tried to pair question difficulty across quizzes (for example, Q3 on the Raft quiz should be about as difficult as Q3 on the Paxos quiz), they are mostly different questions that are hard to compare directly.

7.4.2 Survey

Participants answered three groups of survey questions after taking their second quiz. The questions asked about their prior experience with Paxos and Raft, whether they felt the lectures or quizzes were biased, and which algorithm they felt would be easier to implement or explain. Participants were also asked for open-ended comments or feedback. The full survey and exact questions can be found in Appendix A.3, along with the open-ended comments and feedback.

Many of the Berkeley participants and some of the Stanford participants had prior exposure to Paxos; Figure 7.9 shows their responses to the survey question. At Stanford, 9 of the 31 participants who responded to the question had at least some prior exposure to Paxos. At Berkeley, 6 of the 11 participants who responded to the question had at least some prior exposure to Paxos. No participants reported any prior exposure to Raft (41 participants responded to this question); Raft was still new at the time, so this was expected.

Participants were also asked whether they felt the lectures were of similar quality and whether the quizzes were of similar difficulty; Figure 7.10 shows their responses. 23 of 42 participants responded that the Raft lecture was at least somewhat better than the Paxos lecture, and 20 of 42 participants responded that the Paxos quiz was at least somewhat harder than the Raft quiz. However, these responses may be unreliable: it may have been difficult for participants to separate the intrinsic difficulty of the material or their level of understanding from the lecture quality, or their performance from the quiz difficulty. Therefore, we do not consider this strong evidence against the integrity of our study.
Figure 7.8: CDFs of participants’ scores on individual questions.
Each graph in the figure shows the quiz scores for an individual quiz question. The top row of graphs shows Paxos quiz questions; the bottom row shows Raft quiz questions. The number above each graph is the number of the question (multi-part questions have been aggregated to save space).

The curve in each graph shows a CDF of the data. The y axis is the cumulative fraction of participants who scored up to a given number of points (right/lower is better).

The range of the x axis on each graph corresponds to the point values possible for each question, and one point has the same width in every graph. For example, the graph for a 10-point question is twice as wide as the graph for a 5-point question.

Each graph is also colored to provide summary information at a glance. Each quantile of the data is shaded in a different color, as shown in the legend. Because each graph’s width is scaled to its point value, the size (area) of the shading is proportional to the number of points it represents.
Figure 7.9: Prior Paxos experience survey. Using a five-point scale, participants were asked how much prior exposure they had to Paxos; 42 participants responded to the question. The top graph shows the responses from the Stanford participants, the middle graph shows the responses from the Berkeley participants, and the bottom graph shows the total responses (from all participants).
Figure 7.10: Fairness survey. Using a five-point scale, participants were asked which lecture was better and which quiz was more difficult. 42 participants responded to each question.

Left: Do you think the video lectures were roughly equal in quality, given the nature of the material being presented?

Right: Do you think the quizzes were roughly equal in terms of testing your understanding of the material?

The top graphs show the responses from the Stanford participants, the middle graphs show the responses from the Berkeley participants, and the bottom graphs show the total responses (from all participants).
Figure 7.11: Preferences survey. Using a five-point scale, participants were asked which algorithm would be easier to implement and which would be easier to explain. 41 participants responded to each question.  

**Left:** Suppose you were working at a company and it is your job to implement a replicated state machine. Which algorithm would be easier to implement in a functioning, correct, and efficient system?  

**Right:** Suppose you had to explain either Raft or Paxos to a CS graduate student who hadn’t seen either one previously. Which would be easier to explain?  

The top graphs show the responses from the Stanford participants, the middle graphs show the responses from the Berkeley participants, and the bottom graphs show the total responses (from all participants).
Figure 7.11 shows which algorithms participants felt would be easier to implement or explain. An overwhelming majority of participants preferred Raft for each: 33 of 41 participants reported that Raft would be at least somewhat easier to implement, and 33 of 41 participants reported that Raft would be at least somewhat easier to explain. However, these self-reported feelings may be less reliable than participants’ quiz scores, and participants may have been biased by knowledge of our hypothesis that Raft is easier to understand.

7.5 Discussion about the experimental approach

Initially, we doubted that a user study would be feasible or convincing, but we felt it was the most appropriate way to evaluate Raft’s claim of understandability. Thus, we conducted the user study to provide empirical evidence that Raft is easier to understand than Paxos. Although we consider the study itself successful, it was not particularly well-received by the system community. This section explores whether the study was worth the time and effort we put into it and sheds light into whether this sort of experiment is effective in the systems community.

In our first paper submission on Raft (in 2012), we claimed the Raft algorithm was easier to understand than Paxos, but we had essentially no objective evidence for this. Our anonymous reviewers rightly pointed out this weakness. Excerpts from their reviews serve as evidence that with no evaluation, our claim of understandability was weak:

- It’s not clear Raft is any more understandable than Paxos. While understandability is the key claim to novelty, this claim was not actually evaluated, and may be untrue. . . . I think one thing that would have helped a lot is if the authors chose a concrete “metric of success” and evaluated their system according to that metric. . . . I do like the idea of using understandability as a metric, but that [sic] there was no attempt at all to actually characterize Raft using that metric.

- . . . I understood the algorithm. So, perhaps that speaks to the fact that Raft is indeed understandable.

That said, I do think that you [need] a way to show that Raft is indeed more understandable. A couple of thoughts on doing that:

- Compare implementations of Raft and Paxos using some code complexity measures.
- Explain Raft and Paxos to students, and see which one they understand better. A test of understanding could be writing the pseudocode, a quiz on algorithm behavior, or
extending the algorithm to do something different.

- We encourage the authors to define metrics for “understandability” and systematically explore whether the protocol meets these goals. . . .

Based on this feedback, we conducted the Raft user study and included its results in our second paper submission. Unfortunately, the study did not seem to convince most of our reviewers; they did not seem to find much value in it. Several did not even mention it in their reviews. Others were concerned about the lecture content and quiz questions, even though we referred our readers to the user study lectures and quizzes online. The paper was ultimately accepted (in 2014) after several attempts, but we feel this was despite the reviewers’ generally negative opinions of the study. The following excerpts summarize the reviewers’ opinions about the user study:

- The user study is not that useful.

- The evaluation is thin. Its qualitative thesis (Raft is simpler ⇒ Raft is more understandable ⇒ Raft implementations are more correct) is poorly supported, and hence subject to the reader’s counterintuition. . . . This paper’s evaluation hinges on the user study. Did the tests test the corner cases? Did the students have to prove either system correct, formally?

- I’m disappointed that Section 7 [Clients and log compaction] is empty! Given a choice, I’d rather omit Section 8 [Implementation and evaluation] and include Section 7.

- The user study is a nice idea, but ultimately I don’t think it adds much to the paper. Readers will decide for themselves if the algorithm is understandable or not and the paper’s ultimate impact will depend on this judgement, not a user study.

- The user study is interesting.

  Unclear whether explaining Paxos more clearly would change the results of the user study.

- Reasonable user study about “understandability”.

  User study, while laudable, seems fairly unscientific in the end due to potential large sources of bias.

  I appreciate the author’s attempts to better characterize whether Raft is indeed “more understandable” than Paxos, and care was put into designing the study (e.g., splitting users, presenting them with tests is different orders, etc.). Even so, if our goal is really randomized
trials, the fact that the experimenters wrote the explanations of the two protocols gives me some real pause about some pretty overt bias that could slip into the writeup.

- **User study is fresh and interesting (albeit bias factors are present).**
  The user study is interesting and thought provoking, but it really lacks representativeness both in terms of sample sizes as well as neutrality.

- **I think the understandability study is interesting, but perhaps a little bit of overkill.** Typically researchers can compare two protocols and see for themselves which is simpler . . . But I’m not against a little overkill now and then. However, summarizing a study with a statement such as “students were able to answer questions about Raft 23% better than questions about Paxos” raises immediate questions about whether such figure is very meaningful. In particular 23% seems like a precise figure, but in fact depends a lot on how the tutorials and the test are set up (even when strong measures are taken to ensure fairness, as you have done). Two issues that come to mind are that you can’t ask exactly the same questions about two different protocols, and even if you could, it’s not clear which questions would be the right or fair questions to ask to get at the issue of understandability.

- **The user study based on self-reported understandability scores and correctness of answering problem questions is not particularly convincing.** It would be more convincing if students were made to implement both Paxos and Raft in code, and then compare objectively the time taken, the lines of code, and the overall correctness.
  Concrete suggestion: Please provide some objective measure on understandability, even if it is for a small sample-set of students that implement both Paxos and Raft from just communication primitives.

- **I found the sample size in the section on understandability to be small enough to be worrisome in spite of the good t-test number.** A better test might be the number of unaccounted for failure modes in naïve implementations. I expect Raft to win by a wide margin.

- **Furthermore, a review of the teaching materials in [88] seems to indicate flaws in the way the “Understandability study” was performed.** Some implementations of Multi-Paxos support concurrent proposals which are ordered by the leader. However, it is unclear how Raft does the same. From the description in the paper, I think Raft handles Append entries (performing Append RPCs) sequentially. So, aren’t you comparing the understandability of 2 algorithms with different properties?
• I find the evaluation of using students’ feedback interesting. However, it’s hard to be convinced of your conclusion if I don’t know what quizzes are being asked.

• The authors evaluation of the subjective claim of understandability was done valiantly in section 8.1 [Understandability], bravo.

• We [reviewer and students] didn’t like the user study. It’s not necessary or convincing. Adding more information to make it convincing would not be worth the space cost.

Moreover, conducting the Raft user study was inherently costly and risky in terms of time and effort. Typically, systems papers evaluate their performance quantitatively through machine experiments; such experiments have low cost and fast turnaround. This results in several attractive properties as compared to psychology experiments involving human subjects:

• **Repeatability:** Performance evaluations are typically easy and cheap to repeat. They are often automated so that there is little room for experimenter error in repeating the experiment. On the other hand, psychology experiments require much more human involvement in general, making them more costly and more error-prone to repeat.

• **Iteration:** Easily repeatable experiments make it possible to change the system, its environment, or the experiment based on experimental results. For example, a researcher might discover a bug during performance evaluation, fix it, and rerun the experiments, at little or no additional cost. This can be prohibitively costly in a psychology experiment, as it requires restarting the entire study with a new group of participants.

• **Incremental results:** In a user study, almost all of the work must be done before seeing any results, or even learning whether the basic idea makes sense. This makes such experiments much riskier than performance evaluations, where initial coarse-grained results are often attainable with little effort.

On the other hand, novel approaches can bring some of these properties to human subjects psychology experiments. For example in one study, Dow et al. [24] compare ad impressions using web analytics. This is objective, repeatable, allows iteration, and is incremental. Similar techniques may apply to evaluating understandability in some domains. Massive open online courses (MOOCs) may also be a useful experimental platform for understandability by providing researchers with pools of thousands of students to teach and evaluate.
Despite reviewers’ concerns, we consider the user study to be an essential part of this work. Its results are the only objective evidence we have that Raft is easier to understand than Paxos. The results assume that the study’s lectures are of equal quality and that its quizzes are of equal difficulty. Though we have no way to prove this, the methods aimed to produce equivalent lectures and quizzes, and the materials are available for readers to review. Under this assumption, the results should be convincing, even to skeptical readers.

7.6 Conclusion

The Raft user study compared Raft’s understandability to that of Paxos. The study showed that after learning Paxos or Raft for an hour, students are able to answer equally difficult questions about Raft better than they can about Paxos. We believe we countered all major sources of bias in our study, and the study showed the major result we wanted. However, it took significant time and effort. We hope future techniques, such as leveraging online courses, allow studies to achieve similar results at a lower cost.

The Raft user study was unconventional for systems research, which tends to focus on machine-based performance evaluations. Our study provides substantial evidence in favor of Raft’s understandability, and as far as we know, it is the first study to objectively evaluate consensus algorithms based on teaching and learning. We believe the systems community should carefully consider such studies, as they enable us to advance our collective knowledge through novel kinds of contributions that we could not otherwise convincingly evaluate.
Chapter 8

Correctness

Since the purpose of consensus is to maintain consistency across a replicated state machine, correctness is a key concern. Not only must the algorithm itself be correct, but others must also be able to implement it correctly in real systems. We took a pragmatic approach to correctness in Raft, building a foundation through understanding and intuition, then applying formal methods to the degree they were practical.

To establish the correctness of Raft itself, we developed a formal specification for the basic Raft algorithm and a proof of its safety. These are described in Section 8.1 and can be found in full in Appendix B. Although many other components are needed for a complete system (specifically, membership changes, log compaction, and client interaction), this is an important step towards proving Raft correct. Section 8.2 discusses other methods we tried before arriving at the current proof.

Our goal is for others to be able to build correct systems using Raft, and Section 8.3 describes approaches to doing so. We hypothesize that systems builders will have an easier time developing correct implementations if they fully understand Raft; this is another reason why understandability is so important. We have tried to be clear and precise in describing Raft, but one problem with natural languages is that they can easily be imprecise or ambiguous. Thus, we encourage system builders to compare their understanding with Raft’s formal specification, which is completely precise using mathematical language.
8.1 Formal specification and proof for basic Raft algorithm

We have developed a formal specification and a proof of safety for the consensus mechanism described in Chapter 3; these can be found in Appendix B. The formal specification makes the information summarized in Figure 3.1 completely precise using the TLA+ specification language [50]. It is about 450 lines long and serves as the subject of the proof. It is also useful on its own for anyone implementing Raft.

The formal specification defines the state in a complete Raft cluster with an arbitrary number of servers. It defines an initial state \( \text{Init} \) in which all the servers’ logs are empty and defines all possible transitions from one state to another \( \text{Next} \). There are several such transitions: the network may duplicate or drop a message, and a server may (under the right conditions) receive a message, timeout, restart, become a leader, advance its commit index, receive a request from a client, send a RequestVote request, or send an AppendEntries request. Each transition includes the conditions under which it may occur. For example, a server may only request a vote if it is in the candidate state.

The specification models an asynchronous system (it has no notion of time) with the following assumptions:

- Messages may take an arbitrary number of steps (transitions) to arrive at a server. Sending a message enables a transition to occur (the receipt of the message) but with no particular timeliness.
- Servers fail by stopping and may later restart from stable storage on disk.
- The network may reorder, drop, and duplicate messages.

The formal specification is slightly more general than the Raft algorithm presented in Chapter 3. These differences make the formal specification applicable to a wider range of implementations and also make some of its state transitions more orthogonal, which simplifies the proof. One way in which the formal specification differs from the algorithm’s description is that it uses message-passing rather than RPC. This requires a minor change to the AppendEntries response format, but it eliminates the need to pair responses with requests. The specification also takes more transitions than most implementations would to arrive at the same end state. Since each transition is evaluated atomically, this models smaller atomic regions in an implementation. There are several examples of this:
• When a server times out, it does not grant itself a vote in the same step. Instead, it requests its own vote with an asynchronous RequestVote request message. Also, after a candidate receives its final vote, it becomes leader in a separate transition.

• Leaders do not advance their commit index upon receiving an AppendEntries reply. Instead, they do so in a separate transition. This improves orthogonality, since a leader that forms a single-server cluster can also increase its commit index through the same transition.

• On receiving an AppendEntries request, a server either returns to the follower state, truncates just the last entry from the end of its log, appends just one entry to the end of its log, or replies in one atomic step (it can then continue to process the request in further steps). Reducing this atomic region to just one entry at a time turns out to be important for implementations that write to persistent storage. For example, when entries span multiple files, most file systems would not allow truncating all of the entries atomically. The specification shows that implementations may safely truncate the entries back to front, one or more at a time.

• Servers update their current terms and states upon receiving a message with a larger term, then in a different transition they process the message.

On the other hand, the specification is not as general as possible; that would harm its understandability. For example, some transitions set two variables even when they need not be set atomically.

The proof verifies the State Machine Safety property. It is complete (it relies on the specification alone) and relatively precise (it is about 3,500 words long). The main idea of the proof is summarized in Section 3.6.3. Most of the lemmas (subproofs) show that an invariant holds for all states that are reachable from the initial state in an execution. Using induction, they assume the invariant holds in one state and show that it holds in every possible next state.

It is often necessary in the proof to refer to variables from prior states in the execution. To make this precise, the specification is augmented with history variables; these variables carry information about past events forward to states that follow. For example, one history variable called elections maintains a record of every successful election in the execution, including the complete log of each server at the time it cast its vote and the complete log of the new leader. The history variables are never read in the specification and would not exist at all in a real implementation; they are only “accessed” in the proof.
8.2 Discussion of prior verification attempts

Prior to arriving at the current proof, we tried three other approaches:

1. We first checked an earlier version of Raft in the Murphi model checker [23], which explores the complete state space to check for unsafe conditions. The state space quickly expanded to the point where Murphi could not finish (in a reasonable amount of time), so we had to limit the size of the system to at most four entries in each log, four terms, and five servers. Murphi found one bug in an early version of Raft, which caused log inconsistencies when multiple leaders in a row crashed after incompletely committing log entries. It also missed an important bug in an early version of Raft, where the commitment rule did not account for scenarios like that of Figure 3.7. Murphi most likely missed this bug because of the constrained model size (fortunately, David Mazières found it).

2. We attempted to use the TLA model checker on our specification. We found bugs in creating the specification this way but abandoned this approach because it did not scale well to larger models.

3. We also attempted to use the TLA proof system [21], which introduces a hierarchical language for formally proving properties on TLA specifications and includes a machine checker for such proofs. We mechanically proved the Leader Completeness Property using the TLA proof system, but this proof relied on invariants that have not been mechanically checked (for example, we did not prove the type safety of the specification). Unfortunately, we found it too tedious and time-consuming to use the TLA proof system at the scale of a complete proof. One problem is that TLA is untyped, making it more general but also more tedious [59].

We found the tools for correctness to be limited in various ways. In our experience, model checking was orders of magnitude easier than developing a proof. It essentially requires writing a simplified Raft implementation, and then it can be executed and debugged even easier than a distributed system (model checkers output a full execution trace when any problems occur). Unfortunately, we were not able to verify our models with large enough parameters to be fully convinced of their correctness.

On the other hand, the Raft proof took about six weeks of learning and thinking before any significant progress was made. Creating a proof takes a different skill set from programming and a different sort of creativity. The end result has helped build our confidence in Raft’s safety, but the proof might have bugs. At the scale of the complete Raft specification, only a mechanically checked
proof could definitively be bug-free. We think a machine-checked proof for Raft would be feasible with more capable tools (e.g., Coq [7]), and one has recently been created for Multi-Paxos [101]. However, the time investment required would probably be on the order of several months.

### 8.3 Building correct implementations

There are many possible approaches to building a correct implementation of Raft. The safest approach is to generate an implementation automatically from a proven Raft specification. If the tools are correct, this guarantees that there will be no errors in converting the specification to an implementation. Recent work has shown this to be feasible for Multi-Paxos [101], and we expect it to be for Raft as well. However, this approach has not been very popular in practice so far, perhaps because real-world systems have additional needs, such as performance, that are harder to accommodate in the generated code.

Without generating an implementation, implementers should strive to design their implementations to reduce the possibility of creating bugs, and they should test their implementations to reduce the possibility of encountering bugs in production. The remainder of this section discusses several approaches we think may be effective, though we have not evaluated their effectiveness. Readers may also be interested in the testing strategy used for Chubby [15].

Howard describes a nice design for building ocaml-raft correctly [37, 36]. It collects all the Raft state transitions in one module, while all code for determining when transitions should occur is elsewhere. Each transition checks its pre-conditions using assertions and has no system-level code intermixed, so the code resembles the core of the Raft specification. Because all of the code that manipulates the state variables is collected in one place, it is easier to verify that state variables transition in restricted ways. A separate module invokes the transitions at the appropriate times. Moreover, ocaml-raft can simulate an entire cluster in a single process, which allows it to assert Raft’s invariants across virtual servers during execution. For example, it can check that there is at most one leader per term at runtime.

For end-to-end testing, Jepsen and Knossos are useful tools that have already found bugs in two Raft implementations (in read-only request handling) [45]. Jepsen injects network partitions in a distributed system and determines whether the system loses data. Knossos analyzes clients’ histories of operations against a distributed system to look for ways those histories are not linearizable. Together, these can be used as powerful end-to-end tests for Raft systems.

Some of the most difficult to find bugs are those that only occur in unlikely circumstances such
as during leadership changes or partial network outages. Thus, testing should aim to increase the likelihood of such events. There are three ways to do this.

First, Raft servers can be configured to encourage rare events for testing. For example, setting the election timeout very low and the heartbeat interval very high will result in more leader changes. Also, having servers take snapshots very frequently will result in more servers falling behind and needing to receive a snapshot over the network.

Second, the environment can be manipulated to encourage rare events for testing. For example, servers can be randomly restarted and cluster membership changes can be invoked frequently (or continuously) to exercise those code paths. Starting other processes to contend for servers’ resources may expose timing-related bugs, and the network can be manipulated in various ways to create events that occur only rarely in production, such as:

- Randomly dropping messages (and varying the frequency of drops between servers and links);
- Adding random message delays;
- Randomly disabling and restoring network links; and
- Randomly partitioning the network.

Third, running the tests for a longer period of time will increase the chance of discovering a rare problem. A larger number of machines can run tests in parallel. Moreover, entire clusters can run as separate processes on a single server to reduce network latency, and disk overheads can be reduced by persisting to RAM only (for example, with a RAM-based file system such as tmpfs [64]). While not entirely realistic, these techniques can exercise the implementation aggressively in a much shorter period of time.

### 8.4 Conclusion

We believe our evaluation of Raft’s correctness puts it at least on par with the algorithms used in most Paxos-based systems. Theoreticians have typically proven the safety of only narrow specifications of Paxos, but practitioners deviate from these specifications and extend their systems significantly. The formal specification for Raft is a nearly complete implementation of the basic Raft algorithm presented in Chapter 3, so the fraction of a fully elaborated Raft algorithm that has been proven safe is fairly large. We leave specifying and proving cluster membership, log compaction, and client interaction to future work, along with liveness and availability properties of the basic Raft algorithm.
Chapter 9

Leader election evaluation

This chapter analyzes the performance of leader election in Raft, which occurs when a leader fails and needs to be replaced. Although we expect leader failures to be a rare event, they should be handled in a timely manner. We would like Raft to reliably elect a new leader in a fraction of a second in a typical deployment.

Unfortunately, it is difficult to put a bound on the time or number of messages leader election will take. According to the FLP impossibility result [28], no fault-tolerant consensus protocol can deterministically terminate in a purely asynchronous model. This manifests itself in split votes in Raft, which can potentially impede progress repeatedly during leader election. Raft also makes use of randomized timeouts during leader election, which makes its analysis probabilistic. Thus, we can only say that leader election performs well with high likelihood, and even then only under various assumptions. For example, servers must choose timeouts from a random distribution (they are not somehow synchronized), clocks must proceed at about the same rates, and servers and networks must be timely (or stopped). If these assumptions are not met for some period of time, the cluster might not be able to elect a leader during that period (though safety will always be maintained).

This chapter draws the following conclusions about the performance of Raft’s leader election algorithm:

• When no split vote occurs, elections complete about one third of the way into the election timeout range, on average. They complete slightly faster in clusters with more available servers, since the first server is expected to time out sooner. (Section 9.1)

• Split vote rates are low when the election timeout range is sufficiently broad. We recommend a range that is 10–20 times the one-way network latency, which keeps split votes rates under
40% in all cases for reasonably sized clusters, and typically results in much lower rates. Clusters will experience more split votes as more servers fail, since fewer votes are available. (Section 9.2)

- The number of election terms required to elect a leader follows a geometric distribution, where the expected number is \( \frac{1}{1 - \text{split vote rate}} \). Thus, even a high split vote rate of 50% will only need two election terms on average to elect a leader. A cluster configured with an election timeout that is 10–20 times its one-way network latency will be able to elect a leader in less than 20 times its one-way network latency on average. (Section 9.3)

- Leader election performs well in practice in both local and wide area networks. In a real-world LAN, our system was able to elect a leader in an average of 35 ms when configured with aggressive timeouts, though we suggest using a more conservative timeout range in practice. On a simulated WAN spanning the US, elections typically complete in half a second, and 99.9% of elections complete in 3 seconds, even when two of five servers have failed. (Section 9.4)

- The performance of leader election is not substantially affected by the log comparison in RequestVote RPCs, when some servers will not grant their votes to others. (Section 9.5)

- The basic leader election algorithm can cause disruptions if followers lose connectivity, increment their terms, and then regain connectivity. Section 9.6 extends the basic algorithm with an additional phase to avoid such disruptions.

### 9.1 How fast will Raft elect a leader with no split votes?

The most common case for leader election in Raft is when no split vote occurs, and this section analyzes how long it takes to elect a leader under that assumption. This is expected to be the normal case for Raft clusters; if the cluster is configured correctly, most normal elections will not encounter a split vote. The first server to time out will be able to collect votes from a majority of the cluster and become leader. The timeline of events is shown in Figure 9.1.

With no split votes, the time it takes to elect a leader is determined by how long it takes the first server to time out. The question of when it will time out is illustrated in Figure 9.2. Each server waits for a uniform random timeout after the last time it received a heartbeat. Intuitively, any individual server is expected to time out halfway through the election timeout range, but with more servers it becomes more likely that the first server will time out sooner.
Figure 9.1: Timeline of a typical election when no split vote occurs. The first candidate to
time out successfully collects votes and completes the election (other elections may not be so
fortunate). The figure is drawn to scale assuming the election timeouts are chosen from a range
between 10–20 times the cluster’s one-way network latency.
The “old leader heartbeats” row shows the final heartbeat that the old leader completes, and
when it would have sent its next heartbeats were it not to crash.
The “old leader crash” row shows the interval during which the old leader crashes. This time is
assumed to follow a uniform random distribution within its heartbeat interval. The vertical line
halfway through the interval is its expected (average) value.
The “base election timeout” row shows the interval during which all the followers await addi-
tional heartbeats from the old leader.
The “election timeout range” row shows the interval during which the servers would time out
and start elections to replace the old leader. The vertical lines show expected earliest timeout
values for different numbers of remaining servers (eight, four, and two, respectively).
The “requests for votes” row shows when the candidate sends its RequestVote RPCs to the
other servers and receives their votes.
The “new leader heartbeats” row shows the new leader sending out heartbeat RPCs right away
after becoming leader, then periodically thereafter.

Figure 9.2: What is the smallest random election timeout value chosen by $s$ servers? The
diagram shows random election timeouts a five-server cluster where one server has failed ($s = 4$). $t_{(1)}$ is the smallest timeout value chosen.
We now define the problem more precisely and derive when the first server times out analytically. The variables defined in this chapter are summarized in Table 9.1. Suppose each server chooses its timeouts randomly from the standard uniform distribution (in the range $[0,1]$). Let $T_1 \ldots T_s$ be random variables representing when each of $s$ servers times out. Let $M_s$ be the minimum of $T_1 \ldots T_s$, a random variable representing the time the first server times out. Its cumulative distribution function (CDF) defines the probability that $M_s$ is no greater than a particular time, $t$.

This is equivalent to one minus the probability that all servers times out after $t$:

$$\Pr(M_s \leq t) = 1 - \Pr(M_s > t)$$

$$= 1 - \prod_{i=1}^{s} \Pr(T_i > t)$$

$$= 1 - \prod_{i=1}^{s} (1 - t)$$

$$= 1 - (1 - t)^s$$

For example, consider a cluster with five servers where the prior leader has failed. The probability that the earliest of the remaining four servers times out sometime in the first quarter of the election timeout range is $\Pr(M_s \leq \frac{1}{4}) = 1 - (1 - \frac{1}{4})^4 \approx 0.68$. The CDF is graphed in Figure 9.3 for various values of $s$. 

<table>
<thead>
<tr>
<th>variable</th>
<th>type</th>
<th>meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s$</td>
<td>natural</td>
<td>number of available servers</td>
</tr>
<tr>
<td>$n$</td>
<td>natural</td>
<td>size of full cluster (including unavailable servers)</td>
</tr>
<tr>
<td>$c$</td>
<td>natural</td>
<td>number of servers to time out near each other</td>
</tr>
<tr>
<td>$l$</td>
<td>time</td>
<td>constant half round trip network latency (special case of $L$)</td>
</tr>
<tr>
<td>$L$</td>
<td>random variable of time</td>
<td>half round trip network latency</td>
</tr>
<tr>
<td>$W$</td>
<td>random variable of time</td>
<td>time to write term and vote durably to disk</td>
</tr>
<tr>
<td>$T_i$</td>
<td>random variable of time</td>
<td>timeout of server $i$</td>
</tr>
<tr>
<td>$M_s$</td>
<td>random variable of time</td>
<td>earliest timeout of $s$ servers</td>
</tr>
<tr>
<td>$D_{c,s}$</td>
<td>random variable of time</td>
<td>difference in timeouts of earliest $c$ of $s$ servers</td>
</tr>
<tr>
<td>$E_s$</td>
<td>random variable of time</td>
<td>time to complete an election</td>
</tr>
</tbody>
</table>

Table 9.1: Summary of the variables used throughout this chapter to analyze leader election performance. Times are normalized to the election timeout range (ranging from 0 to 1).
The graph shows the probability that the earliest server times out before $t$ when different numbers of servers are available. The point on each line shows the time when the first server is expected to time out ($E[M_s]$).

The probability density function (PDF) of $M_s$ is the derivative of the CDF:

$$f_{M_s}(t) = \frac{d}{dt} \Pr(M_s \leq t)$$

$$= \frac{d}{dt} (1 - (1 - t)^s)$$

$$= -\frac{d}{dt} (1 - t)^s$$

$$= s(1 - t)^{s-1}$$

The expected value (mean) of $M_s$ is calculated from the PDF:

$$E[M_s] = \int_0^1 t f_{M_s}(t) \, dt$$

$$= \int_0^1 t(s(1 - t)^{s-1}) \, dt$$

$$= \frac{(1 - t)^s(s t + 1)}{s + 1} \bigg|_{t=0}^{t=1}$$

$$= \frac{1}{s + 1}$$
For example, with four available servers, the first timeout is expected to occur \( \frac{1}{5} \) of the way through the election timeout range. Fortunately, this very simple expression is a good estimate of Raft’s overall election performance, since elections complete soon after the first candidate times out when no split vote occurs.

More precisely, if there is no split vote, the full election requires a candidate to time out and request votes, once the leader crashes:

\[
E_s = \text{baseline election timeout} + M_s + \text{time to request votes} - \text{heartbeat adjustment}
\]

\[
E_s = 1 + M_s + 2L + W - U(0, \frac{1}{2})
\]

\[
E[E_s] = 1 + \frac{1}{s + 1} + 2E[L] + E[W] - \frac{1}{4}
\]

where election timeouts are chosen from the range \([1, 2]\), \(L\) is the network latency, and \(W\) is the time to write the votes persistently to disk. A uniform random time value from the range \([0, \frac{1}{2}]\) is subtracted, since leaders are expected to crash randomly within their heartbeat intervals rather than immediately after sending heartbeats.

### 9.2 How common are split votes?

The previous section analyzed the performance of normal elections when no split vote occurs. In practice, two or more candidates may time out at similar times, leading to split votes. Split votes cause additional election timeout delays, and if they occur too frequently, they can impact election performance dramatically. This section first analyzes split votes under a simplifying assumption that network latencies are constant, then subsequently relaxes this assumption.

#### Split vote rate with fixed latency

Split votes can be calculated more simply if network latencies are fixed. Let the constant \(l\) be the one-way latency between any two servers in the cluster, measured as a fraction of the election timeout range. Because of the fixed network latency, the first server to time out is guaranteed to get the votes of all servers that don’t time out within \(l\) of it, and it will receive none of the votes of the other servers, who will each vote for themselves. The probability of a split vote is thus the probability that too many candidates time out within \(l\) of each other. For example, consider a five-server cluster in which only four servers are available. As illustrated in Figure 9.4, if only two servers time out
CHAPTER 9. LEADER ELECTION EVALUATION

Figure 9.4: These examples show two similar elections in a five-server cluster when one server has failed and network messages have a fixed latency. Each server’s random timeout value is shown on the timelines, where \( t_1 \) is the smallest value chosen, \( t_2 \) is the second-smallest, and so on. In the top election, the first server is able to collect votes from itself, the third server, and the fourth server. However, in the bottom election, its RequestVote RPC cannot reach the third server in time before that server times out; thus, the election ends in a split vote.

Within \( l \) of each other, the earliest server will be able to collect votes from itself and the other two servers and become leader. However, if three servers time out within \( l \), then the earliest server will only be able to reach one other server in time to receive its vote, so the vote is split.

To derive a general formula for when split votes occur, let \( c \) denote the number of servers that time out within \( l \) of each other and let \( n \) be the size of the full cluster. The first server will get its own vote plus votes from the \( s - c \) servers that time out at least \( l \) time after the first. Thus, a split vote occurs when the following condition holds:

\[
\text{votes needed} > \text{votes available to earliest server} = \left\lfloor \frac{n}{2} \right\rfloor + 1 > 1 + (s - c) \\
\left(\text{or equivalently} \right) \quad c > s - \left\lfloor \frac{n}{2} \right\rfloor
\]

How often split votes occur thus depends on how often at least \( c \) servers timeout within \( l \) of each other. Let \( D_{c,s} = T(c) - T(1) \), where \( T(i) \) is a random variable representing the timeout of the \( i \)-th of \( s \) servers in sorted order; \( D_{c,s} \) is the time after the first server times out that the \( c \)-th server times out. The probability of split votes is then \( \Pr(D_{c,s} < l) \), where \( c \) is determined by the formula given above \( (s - \left\lfloor \frac{n}{2} \right\rfloor + 1) \).

We now derive the CDF for \( D_{c,s} \), denoted \( \Pr(D_{c,s} \leq l) \). Suppose the first server times out at \( t \).
First, if $t < 1 - l$, each of the following servers times out within $l$ time after $t$ with probability $l^{1-t}$.

The probability that the second through $c^{th}$ servers time out within $l$ time after $t$, and the remaining $s - c$ servers do not, is given by:

$$\left(\frac{s-1}{c-1}\right) \left(\frac{l}{1-t}\right)^{c-1} \left(\frac{1-t-l}{1-t}\right)^{s-c}$$

Instead, if $t \geq 1 - l$, then any server that times out after the first must time out within $l$ time of $t$.

Thus, all $s$ servers will time out within $l$ of the first with probability 1, and the probability that any server does not timeout within $l$ of the first is 0. Putting this together, we can now derive the CDF:

$$\Pr(D_{c,s} \leq l) = \sum_{k=c}^{s} \left( \int_{0}^{1-l} \Pr(\text{exactly } k \text{ servers time out in } t \text{ to } (t+l) \text{ range } | M_s = t) f_{M_s}(t) \, dt + \int_{1-l}^{1} f_{M_s}(t) \, dt \right)$$

$$= \sum_{k=c}^{s} \left( \int_{0}^{1-l} \left(\frac{s-1}{k-1}\right) \left(\frac{l}{1-t}\right)^{k-1} \left(\frac{1-t-l}{1-t}\right)^{s-k} f_{M_s}(t) \, dt \right) + \int_{1-l}^{1} f_{M_s}(t) \, dt$$

$$= \sum_{k=c}^{s} \left( \int_{0}^{1-l} \left(\frac{s-1}{k-1}\right) \left(\frac{l}{1-t}\right)^{k-1} \left(\frac{1-t-l}{1-t}\right)^{s-k} s(1-t)^{s-1} \, dt \right) + \int_{1-l}^{1} s(1-t)^{s-1} \, dt$$

$$= \sum_{k=c}^{s} \left( \frac{s-1}{k-1} \right) \left( \frac{s}{s-k+1} \right) \left(\frac{l}{1-t}\right)^{k-1} \left(\frac{1-t-l}{1-t}\right)^{s-k} \left|_{t=0}^{1-l} \right| + \left( - (1-t)^{s} \right) \bigg|_{t=1-l}^{1}$$

$$= \sum_{k=c}^{s} \left( \frac{s-1}{k-1} \right) \frac{s}{s-k+1} \left(\frac{l}{1-t}\right)^{k-1} \left(\frac{1-t-l}{1-t}\right)^{s-k+1} \right) + l^{s}$$

$$= \left( \sum_{k=c}^{s} \frac{(s-1)!}{(s-k)!(s-k+1)!} \left(\frac{l}{1-t}\right)^{k-1} \left(\frac{1-t-l}{1-t}\right)^{s-k+1} \right) + l^{s}$$
= \left( \sum_{k=c}^{s} \frac{s!}{(k-1)! (s-k+1)!} l^{k-1} (1-l)^{s-k+1} \right) + l^s

= \left( \sum_{k=c}^{s} \frac{s^k}{(s-k)!} l^{k-1} (1-l)^{s-k+1} \right) + l^s

= \left( \sum_{k=c-1}^{s-1} \frac{s^k}{k!} l^{k} (1-l)^{s-k} \right) + l^s

= \sum_{k=c-1}^{s} \frac{s^k}{k!} l^{k} (1-l)^{s-k}

(The CDF somewhat resembles a binomial distribution, which hints that there may exist an easier derivation.)

For example, consider a five-server cluster with four available servers \((s = 4)\). A split vote will occur if the earliest three servers time out within \(l\) of each other \((c = 3)\). If the election timeout range is 100 ms, the earliest three servers will time out within \(l\) of each other, resulting in a split vote:

- In about 0.06\% of elections, if the one-way network latency is 1 ms, \(\Pr(D_{3,4} \leq 0.01)\);
- In about 5.2\% of elections, if the one-way network latency is 10 ms, \(\Pr(D_{3,4} \leq 0.1)\); and
- In about 18.1\% of elections, if the one-way network latency is 20 ms, \(\Pr(D_{3,4} \leq 0.2)\).

Figure 9.5 graphs the CDF formula for a range of cluster sizes. The first thing to observe is that failures have a very large effect on split vote rates, especially if the cluster is down to a bare majority of its original members. For example, a five-server cluster with \(l = 0.2\) will encounter less than 20\% split votes after one failure; if the same cluster encounters a second failure, about half of election terms will encounter split votes. To prepare for worst-case scenarios, the election timeout range should be set to produce tolerable values when a bare majority of the cluster is available.

Second, larger clusters experience fewer split votes with the same number of failures, but they experience an even larger worst-case split vote rate as a result of being able to tolerate more failures. For example, a nine-server cluster with \(l = 0.2\) will experience only about a 15\% rate of split votes after two failures (compare with 50\% for a five-server cluster). However, when it is down to its bare majority with four failures, the nine-server cluster will experience a nearly 70\% split vote rate.

Third, keeping the number of available servers constant, larger full cluster sizes will have more split votes. For example, with \(l = 0.2\), a nine-server cluster with six available servers will experience about a 35\% rate split votes; a seven-server cluster with six available servers will experience only
Figure 9.5: The graphs show the likelihood of split votes for various cluster sizes and numbers of server failures, given a fixed network latency $l$. Each graph shows a different full cluster size, and the curves on each graph show different numbers of failed servers in that cluster. Each value represents the likelihood that a split vote will occur because the first $c$ of the $s$ servers timed out within $l$ of each other, where $c$ is determined by $s - \left\lfloor \frac{n}{2} \right\rfloor + 1$. For example, a five-server cluster with two failures and $l = 0.2$ will see about half of elections end in split votes.
about a 10% rate split votes. This is because larger full clusters require more votes to win an election; fewer candidates need to time out within $l$ of each other in order to produce a split vote.

Finally, the graphs suggest that choosing an election timeout range of 10–20 times the one-way network latency (so $l = 0.1$) will result in low split vote rates in all clusters, assuming network latencies are nearly constant. With this setting, a nine-server cluster that has experienced four failures will encounter 40% split votes, and most typical clusters will encounter much fewer. Smaller election timeouts (larger $l$ values) may also work in many deployments, but they should be tested more carefully to make sure.

**Split vote rate with variable latency**

When network latency is variable, calculating split vote rates is more complicated. The problem is that a RequestVote message sent by one server can overtake a RequestVote message sent earlier by a different server. Thus, the first server to time out is no longer guaranteed to collect all of the votes of servers that do not vote for themselves. The first server is still the most likely candidate to win, by virtue of sending requests for votes first, but its advantage depends on how much earlier it timed out. Thus, with variable latency, we intuitively expect somewhat higher rates of split votes (there is more competition).

Rather than model this mathematically, we used a small simulation. Each run followed the following steps (before optimization):

1. Assign random timeouts to each of $s$ servers.
2. If a server has not voted by the time it times out, it votes for itself and schedules RequestVote messages to be delivered to other servers after random latencies.
3. If any server collects a majority of votes, the election term is considered successful; otherwise, it is considered a split vote.

After 10,000 runs, the fraction of split votes was calculated.

Figure 9.6 shows the split vote rates for messages with uniform random latencies in the range $[l_{\min}, l_{\max}]$, as calculated by the simulation. (Uniform random latencies may not be a realistic distribution, but they are the simplest case and can help with estimating more complex distributions.) The overall conclusion from these graphs is the same as with fixed latency: more failures result in significantly higher split vote rates.
Figure 9.6: The contour graphs show split vote rates for various cluster sizes and numbers of failed servers. Each narrow contour line denotes a 1% increase in the probability of split votes; each medium contour line denotes a 10% increase; the thick contour line visible in some graphs denotes the 50% barrier. Split votes are always 0% at the origin, where messages are instantaneous. The points on the x axis, where the latency range is zero, correspond to the split vote rates with fixed latencies in Figure 9.5.
For example, the probability of a split vote for a five-server cluster after one failure can be found in the graph in the second column and second row. When latencies are chosen randomly and uniformly between 0.1 and 0.2, the point with a minimum latency of 0.1 and a latency range of 0.1 reveals, by counting contour lines, that the split vote rate is about 16%. With two failures, the probability of a split vote in the same cluster is nearly 40%.
In clusters with only a bare majority of servers available (the bottom graph in each column), the contour lines are very linear with a slope of about \(-2\): they have about the same split vote rate when keeping the average network latency constant. This indicates that the split vote rates for bare-majority clusters can be accurately approximated with our fixed latency model using the average of the variable latency range. For example in a nine-server cluster with four failures, a variable latency chosen randomly from the range \([0.1, 0.2]\) results in a similar split vote rate as a fixed latency of 0.15.

In clusters with fewer failures, the contour lines aren’t always linear, and they typically have less slope in general (they are flatter). For example, in a five-server cluster with no failures, a variable latency between 0.1 and 0.2 has a similar split vote rate as a fixed latency of about 0.2 (the slope of the contour lines is only about \(-1\)). Typically, split vote rates can be bounded with our fixed latency model using the maximum of the variable latency range. This is true for about 78\% of the data points shown in the figure; however, this approximation works least well for large clusters with few failures, as these contour lines are most curved.

9.3 How fast will Raft elect a leader when split votes are possible?

Given a split vote rate, we can estimate the total election time. Raft will elect a leader as soon as an election term successfully completes without a split vote. When a split vote occurs, it’s likely that all servers have reset their timers, since servers do this when they grant a vote (this isn’t quite true when logs differ; see Section 9.5). Thus, the next election term has the same probability of success as an entirely new election and will take just as long. In other words, each election term is essentially memoryless, and the number of election terms required in an election can be modeled as a geometric distribution, where the probability of success is the probability that a split vote does not occur. Therefore, Raft elections are expected to complete in \(\frac{1}{1 - \text{split vote rate}}\) election terms on average.

If a split vote occurs in a particular election term, the election term takes about \(1 + M_z\) time units plus a one-way network latency to reset the server’s election timers. We do not include the time for the candidate to record its own vote on disk, since this time can be overlapped with the RequestVote messages (with this optimization, the candidate may not count its own vote towards leadership until the vote is durably recorded). After the vote is split, the cluster must wait another election timeout before the next election term begins. This repeats for each split vote, then the time for an election...
with no split votes (from Section 9.1) is additional. Thus, the total time for an election, $E_s$, is:

$$E_s = \left( \sum \text{time for split vote} \right) + \left( \text{time for election with no split vote} \right)$$

$$E_s = \left( \sum \text{split votes} \right) \left( 1 + M_s + L \right) + \left( 1 + M_s + 2L + W - U(0, \frac{1}{2}) \right)$$

$$\mathbb{E}[E_s] = \left( \frac{1}{1 - \text{split vote rate}} - 1 \right) \times \left( 1 + \frac{1}{s + 1} + \mathbb{E}[L] \right) + \left( 1 + \frac{1}{s + 1} + 2\mathbb{E}[L] + \mathbb{E}[W] - \frac{1}{4} \right)$$

$$\mathbb{E}[E_s] = \frac{1}{1 - \text{split vote rate}} \times \left( 1 + \frac{1}{s + 1} + \mathbb{E}[L] \right) + \mathbb{E}[L] + \mathbb{E}[W] - \frac{1}{4}$$

where $L$ is the one-way network latency and $W$ is the latency for a durable disk write.

Howard [37] suggests an optimization to decrease the time for an election after split votes occur. The optimization separates followers’ timeouts from candidates’ timeouts, where candidates select smaller timeouts from a distribution with a smaller range. This results in faster iterations once split votes have occurred, though it risks additional split votes. The remainder of this chapter does not use this optimization.

Figure 9.7 plots the expected time to elect a leader when the network latency is fixed, by combining the formula for $\mathbb{E}[E_s]$ with the formula for $Pr(D_{c,s} \leq l)$. From the graphs, a Raft cluster with a sufficiently broad timeout range will usually elect a leader within 20 times the one-way network latency, even when running with a bare majority of available servers. This suggests that most datacenter Raft deployments should be able to achieve typical leader election times under 100 ms. Even worst case global deployments, with one-way latencies of 200 ms, should be able to typically elect leaders within 4 seconds. (Election times may be larger if some servers are deployed on other planets.)

Each of the curves has a knee. If the timeout range is chosen to be too short, too many servers time out before others are able to collect votes, resulting in poor election times. Once timeout ranges are sufficiently large (about 3–8 times the network latency, depending on the cluster), the curves become linear with a slight upward slope: elections complete after few or no split votes, but they must wait longer for each timeout to elapse.

The graphs provide insight into how to configure election timeouts: a conservative setting is probably best in practice. The minimum point on the graphs represents the best average election time possible for each given cluster configuration. However, attaining this minimum time is quite risky, since the minimum is close to the knee in the curve. If the network latency turns out to be slightly higher than anticipated in practice, that might push the system into the left region of the
Figure 9.7: The expected total election times for various clusters, as defined by $E[E_s]$, with a fixed one-way network latency. It excludes the time to write to stable storage (which is usually negligible). The timeout range and expected overall election time are presented as multiples of the one-way network latency ($l$), since $l$ is typically fixed in a given deployment.
code: LogCabin [86], written in C++11
OS: x86-64 RHEL6 (Linux 2.6.32)
CPU: Xeon X3470 (4 cores, 8 hyperthreads)
disk: ext4 file system on Crucial M4 SSDs (1 SSD per server)
network: Protocol Buffers [111] over TCP/IP over 1 gigabit Ethernet
configuration: in-memory state machine, no log compaction

Table 9.2: Experimental setup for real-world LAN benchmark.

graph where election times skyrocket. It is better to configure systems farther to the right, trading off a slightly higher average election time in exchange for a more robust system. Thus, we recommend using a timeout range that is ten times the one-way network latency (even if the true network latency is five times greater than anticipated, most clusters would still be able to elect a leader in a timely manner).

9.4 How fast will the complete Raft algorithm elect a leader in real networks?

The previous sections were based on simplified models of how leader election works in Raft. We wanted to know how fast Raft will be able to elect a leader in the real world. To find out, this section evaluates Raft’s leader election algorithm using a real-world benchmark in a LAN environment and a realistic simulator in a slower WAN environment.

Real-world implementation on a LAN

We used LogCabin to measure the performance of Raft’s leader election algorithm on five servers connected by a gigabit Ethernet network. The experimental setup is summarized in Table 9.2. The benchmark repeatedly crashed the leader of a cluster of five servers and timed how long it took to detect the crash and elect a new leader. The benchmark measured the time from when the old leader crashed until the other servers received the new leader’s first heartbeat (see Figure 9.1). The leader was crashed randomly within its heartbeat interval, which was half of the minimum election timeout for all tests. Thus, the smallest possible downtime was about half of the minimum election timeout.

The benchmark tried to generate a worst-case scenario for leader election. First, it synchronized the old leader’s heartbeat RPCs before causing the old leader to exit; this made the follower’s election timers start at approximately the same time, leading to many split votes if the timeout values
(a) Time to elect new leader when varying the range of randomness in election timeouts.

(b) Time to elect new leader when scaling the minimum election timeout.

**Figure 9.8:** The graphs show the time to detect and replace a crashed leader in the real-world LAN benchmark. Each line represents 1,000 trials (except for 100 trials for “150–150 ms”) and corresponds to a particular choice of election timeouts; for example, “150–155 ms” means that election timeouts were chosen randomly and uniformly between 150 ms and 155 ms. The steps that appear on the graphs show when split votes occur (the cluster must wait for another election timeout before a leader can be elected). The measurements were taken on a cluster of five servers with a broadcast time (network round trip plus disk write) of roughly 15 ms. Results for a cluster of nine servers are similar.
were not sufficiently randomized. Second, the servers in each trial had different log lengths, so two of the four servers were not eligible to become leader (however, Section 9.5 will show that this has only a minor effect on election times).

Figure 9.8(a) shows that elections complete in under one second when the timeout range is sufficiently broad. A small amount of randomization in the election timeout is enough to avoid split votes in elections. In the absence of randomness, leader election consistently took longer than 10 seconds due to many split votes. Adding just 5 ms of randomness helps significantly, resulting in a median downtime of 287 ms. Using more randomness improves worst-case behavior: with a 50 ms random range, the worst-case completion time (over 1,000 trials) was 513 ms.

Figure 9.8(b) shows that downtime can be reduced by reducing the election timeout. With an election timeout of 12–24 ms, it takes only 35 ms on average to elect a leader (the longest trial took 152 ms). However, lowering the timeouts beyond this point violates Raft’s timing requirement: leaders have difficulty broadcasting heartbeats before other servers start new elections. This can cause unnecessary leader changes and lower overall system availability. We recommend using a conservative election timeout such as 150–300 ms; such timeouts are unlikely to cause unnecessary leader changes, result in a low rate of split votes, and will still provide good availability.

**Simulated WAN network**

We developed a simulator called AvailSim [85] to explore a wider range of leader election scenarios. Unlike the fixed network in our real-world test cluster, AvailSim allows the latency of the simulated network to be configured arbitrarily. (We used AvailSim to interactively explore a wide space of leader election scenarios and algorithms, but this chapter only includes a few relevant results.)

AvailSim is a close approximation to a complete Raft system, but its election time results differ from real elections in two ways:

1. Each server in AvailSim begins with a fresh election timer. In practice, the leader will crash at some random point in time between heartbeats. The election times produced by AvailSim are thus an average of half a heartbeat interval too large.

2. AvailSim does not add any time for processing messages or writing to disk (these are infinitely fast in the simulator). CPU time should be short relative to network latency, and disks need not play a significant role in leader election anyhow (see Section 9.3).

We used AvailSim to approximate a WAN spanning the continental US. Each message was assigned a latency chosen randomly from the uniform range of 30–40 ms, and the servers’ election
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Figure 9.9: Election performance as calculated by AvailSim for a WAN (one-way network latency of 30–40 ms). The figure shows a cluster of five servers with zero, one, and two servers having failed.

The left graph plots the CDFs of election times. The right graph plots the same curves on a reverse-logarithmic y axis to magnify detail on the tail of the distribution. Each CDF summarizes 10,000 simulated elections. The point on each curve marks the average election time.

Figure 9.10: Election performance as calculated by AvailSim when each server has a different log (using the same WAN configuration as Figure 9.9). Performance is similar to Figure 9.9, where the servers’ logs are all the same.
timeout range was set accordingly to 300–600 ms (about 10–20 times the one-way network latency).

Figure 9.9 shows how quickly a five-server cluster elects a leader in this WAN environment. When only one of the five servers has failed, the average election completes within about 475 ms, and 99.9% of elections complete within 1.5 s. Even when two of the five servers have failed, the average election takes about 650 ms (about 20 times the one-way network latency), and 99.9% of elections complete in 3 s. We believe these election times are more than adequate for most WAN deployments.

9.5 What happens when logs differ?

Most of this chapter has assumed that servers grant their votes on a purely first-come-first-served basis. In reality, Raft restricts how servers may grant votes: the RequestVote RPC contains information about the candidate’s log, and a voter does not grant its vote or reset its election timer if the voter’s log is more up-to-date than the candidate’s.

We used AvailSim to investigate what effect, if any, this voting restriction has on leader election performance. The simulation was configured with the same WAN network as in Section 9.4, but each server was configured with a different log. Thus, only three, two, or one of the five servers were eligible to become leader, depending on whether zero, one, or two of the servers had failed.

Figure 9.10 shows the results; performance is very similar to when the servers had equal logs. The curves do have slightly different shapes (they have sharper corners), but the effect is small. Thus, we do not believe the log comparison adversely affects leader election performance.

9.6 Preventing disruptions when a server rejoins the cluster

One downside of Raft’s leader election algorithm is that a server that has been partitioned from the cluster is likely to cause a disruption when it regains connectivity. When a server is partitioned, it will not receive heartbeats. It will soon increment its term to start an election, although it won’t be able to collect enough votes to become leader. When the server regains connectivity sometime later, its larger term number will propagate to the rest of the cluster (either through the server’s RequestVote requests or through its AppendEntries response). This will force the cluster leader to step down, and a new election will have to take place to select a new leader. Fortunately, such events are likely to be rare, and each will only cause one leader to step down.

If desired, Raft’s basic leader election algorithm can be extended with an additional phase to
prevent such disruptions, forming the Pre-Vote algorithm. In the Pre-Vote algorithm, a candidate only increments its term if it first learns from a majority of the cluster that they would be willing to grant the candidate their votes (if the candidate’s log is sufficiently up-to-date, and the voters have not received heartbeats from a valid leader for at least a baseline election timeout). This was inspired by ZooKeeper’s algorithm [42], in which a server must receive a majority of votes before it calculates a new epoch and sends NewEpoch messages (however, in ZooKeeper servers do not solicit votes, other servers offer them).

The Pre-Vote algorithm solves the issue of a partitioned server disrupting the cluster when it rejoins. While a server is partitioned, it won’t be able to increment its term, since it can’t receive permission from a majority of the cluster. Then, when it rejoins the cluster, it still won’t be able to increment its term, since the other servers will have been receiving regular heartbeats from the leader. Once the server receives a heartbeat from the leader itself, it will return to the follower state (in the same term).

We recommend the Pre-Vote extension in deployments that would benefit from additional robustness. We also tested it in various leader election scenarios in AvailSim, and it does not appear to significantly harm election performance.

9.7 Conclusion

Raft’s leader election algorithm performs well in a wide variety of scenarios. It is able to elect leaders within tens of milliseconds on average on a real-world LAN. When election timeouts are chosen randomly from a range of 10–20 times the one-way network latency, leaders are elected within about 20 times the one-way network latency on average. Tail election times are also fairly short. For example, 99.9% of elections complete in less than 3 seconds when the one-way network latency is as high as 30–40 ms.

This chapter answered most of the basic questions about how Raft’s leader election algorithm performs. Further analysis is required to answer the following additional questions:

- How much longer does leader election take when servers start with different initial current term numbers?
- How does leader election perform in asymmetric networks, where each link has a different latency?
- How well does leader election work on networks with severe packet loss?
- How well does leader election work when servers experience severe clock drift?
Another interesting area of research would be to explore setting election timeouts dynamically. Raft’s leader election performance depends on a properly configured election timeout, and it would be nice to configure this election timeout automatically and dynamically. However, we do not know how leader election will perform if different servers use different election timeout ranges (this is related to the clock drift question above).
Chapter 10

Implementation and performance

This chapter discusses Raft’s implementations and its performance for log replication.

10.1 Implementation

We have implemented Raft as part of LogCabin, a replicated state machine implemented as a network service. We initially developed LogCabin to store configuration information for RAMCloud [90] and assist in failover of the RAMCloud coordinator. We had planned to implement Paxos in LogCabin, but the difficulties we faced motivated us to develop Raft. LogCabin then served as our test platform for new ideas in Raft, and also as a way to verify that we understood the issues of building a complete and practical system. The Raft implementation in LogCabin contains roughly 2,000 lines of C++ code, not including tests, comments, or blank lines. The source code is freely available [86]. Its architecture is discussed in the next section.

In addition to LogCabin, there are dozens of third-party open-source implementations of Raft in various stages of development [92]. Many of these use different architectures than LogCabin, such as the actor model [106, 73, 68] or event-based programming [75, 99, 107]. Various companies are also deploying Raft-based systems [92]. For example, Facebook is currently testing HydraBase, a fork of Apache HBase [3] that uses Raft for replication [29].

10.1.1 Threaded architecture

Raft lends itself to a straightforward implementation architecture using threads, as shown in Figure 10.1. This is not the only possible architecture, but it is the approach we have taken in LogCabin.
Figure 10.1: In LogCabin, consensus state for each server is stored in a monitor protected by a single lock, accessed by a collection of threads. The threads communicate with other servers (“peer threads”), handle incoming requests from clients and other servers (“service threads”), execute commands in the state machine (“state machine thread”), implement timeouts (“timer threads”), and write log entries to disk (“log sync thread”).

Each server consists of a collection of shared state variables managed in a monitor style with a single lock. Five groups of threads call into the monitor to manipulate the state:

- **Peer threads**: There are as many peer threads as there are other servers in the cluster; each peer thread manages the RPCs to one of the other servers. Each thread enters the consensus state monitor, using a condition variable to wait for events that require communication with the given server. Then it leaves the monitor (releasing the lock) and issues an RPC. Once the RPC completes (or fails), the peer thread reenters the consensus state monitor, updates state variables based on the RPC, and waits for the next event that requires communication.

- **Service threads**: Several threads handle incoming requests from clients and other servers. These threads wait outside the consensus state monitor for incoming requests, then enter the monitor to carry out each request.

- **State machine thread**: One thread executes the state machine. It enters the consensus state monitor to wait for the next committed log entry; when an entry is available, it leaves the monitor, executes the command, and returns to the monitor to wait for the next command.

- **Timer threads**: One thread manages the election timer for both followers and candidates; it starts a new election once a randomized election timeout has elapsed. A second thread causes the server to return to the follower state if, as leader, it is unable to communicate with a majority of the cluster; clients are then able to retry their requests with another server (see Section 6.2).
• **Log sync thread:** When the server is leader, one thread writes log entries durably to disk. This is done without holding the lock on the consensus state, so replication to followers can proceed in parallel; see Section 10.2.1. For simplicity, followers and candidates write directly to disk from their service threads while holding the consensus lock; they do not use the log sync thread.

## 10.2 Performance considerations

Raft’s performance is similar to other consensus algorithms such as Multi-Paxos. The most important case for performance is when an established leader is replicating new log entries. Raft achieves this using the minimal number of messages (a single round-trip from the leader to half the cluster). It is also possible to further improve Raft’s performance. For example, Raft easily supports batching and pipelining requests for higher throughput and lower latency, as described below. Chapter 11 discusses various other optimizations that have been proposed in the literature for other algorithms; many of these could be applied to Raft, but we leave this to future work.

Figure 10.2(a) shows the steps Raft must take to process a client’s request. Typically, the most time-consuming steps are writing the new log entry to disk and replicating it across the network. Writing to disk can take anywhere from 100 µs for a fast solid state disk to 10 ms for a slow magnetic disk, while the latencies of today’s networks can vary from 5 µs round trip times in highly optimized datacenter networks to 400 ms round trip times for networks that span the globe. In our experiments on a local area network, either the disk or the network dominated, depending on which model of solid state disk we used.

### 10.2.1 Writing to the leader’s disk in parallel

One useful performance optimization can remove a disk write from Raft’s critical path. In a naïve implementation, the leader writes the new log entry to disk before replicating the entry to its followers. Then, the followers write the entry to their disks. This results in two sequential disk writes on the path to process a request, contributing significant latency for deployments where disk writes are a dominant factor.

Fortunately, the leader can write to its disk in parallel with replicating to the followers and then writing to their disks; see Figure 10.2(b). To handle this simply, the leader uses its own *match index* to indicate the latest entry to have been durably written to its disk. Once an entry in the leader’s current term is covered by a majority of match indexes, the leader can advance its commit index.
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(a) Unoptimized Raft pipeline.

(b) Optimized Raft pipeline.

Figure 10.2: To process a client’s request in an unoptimized implementation of Raft, the leader takes the following steps, shown in (a): it receives the client’s request, appends it to its local log, flushes the log entry to disk, and sends out AppendEntries requests. Then, the followers append the entry to their logs and flush it to their disks. Once the leader receives positive AppendEntries responses from half of its followers, it marks the entry committed, applies the entry to its state machine, and replies to the client. In (b), the leader writes the log entry to its disk in parallel with replicating the entry to the followers, which can reduce latency significantly.

The leader may even commit an entry before it has been written to its own disk, if a majority of followers have written it to their disks; this is still safe. LogCabin implements this optimization.

10.2.2 Batching and pipelining

Raft supports batching and pipelining of log entries, and both are important for best performance. Many of the costs of request processing are amortized when multiple requests are collected into a batch. For example, it is much faster to send two entries over the network in one packet than in two separate packets, or to write two entries to disk at once. Thus, large batches optimize throughput and are useful when the system is under heavy load. Pipelining, on the other hand, optimizes latency under moderate load by allowing one entry to start to be processed when another is in progress. For example, while a follower is writing the previous entry to disk, pipelining allows the leader to replicate the next entry over the network to that follower. Even at high load, some amount of pipelining can increase throughput by utilizing resources more efficiently. For example, a follower needs to receive entries over the network before it can write them to disk; no amount of batching can use both of these resources at once, but pipelining can. Pipelining also works against batching to some degree. For example, it might be faster overall to delay requests and send one big batch to
followers, rather than pipelining multiple small requests.

Batching is very natural to implement in Raft, since AppendEntries supports sending multiple consecutive entries in one RPC. Leaders in LogCabin send as many entries as are available between the follower’s next index and the end of the log, up to one megabyte in size. The one megabyte limit is arbitrary, but it is enough to use the network and disk efficiently while still providing frequent heartbeats to followers (if one RPC got to be too large, the follower might suspect the leader of failure and start an election). The follower then writes all the new entries from a single AppendEntries request to its disk at once, thus making efficient use of its disk.

Pipelining is also well-supported by Raft. The AppendEntries consistency check guarantees that pipelining is safe; in fact, the leader can safely send entries in any order. To support pipelining, the leader treats the next index for each follower optimistically; it updates the next index to send immediately after sending the previous entry, rather than waiting for the previous entry’s acknowledgment. This allows another RPC to pipeline the next entry behind the previous one. Bookkeeping is a bit more involved if RPCs fail. If an RPC times out, the leader must decrement its next index back to its original value to retry. If the AppendEntries consistency check fails, the leader may decrement the next index even further to retry sending the prior entry, or it may wait for that prior entry to be acknowledged and then try again. Even with this change, LogCabin’s original threading architecture still prevented pipelining because it could only support one RPC per follower; thus, we changed it to spawn multiple threads per peer instead of just one.

This approach to pipelining works best if messages are expected to be delivered in order in the common case, since reordering may lead to inefficient retransmissions. Fortunately, most environments will not reorder messages often. For example, a leader in LogCabin uses a single TCP connection to each follower, and it only switches to a new connection if it suspects a failure. Since a single TCP connection masks network-level reordering from the application, it is rare for LogCabin followers to receive AppendEntries requests out of order. If the network were to commonly reorder requests, the application could benefit from buffering out-of-order requests temporarily until they could be appended to the log in order.

The overall performance of a Raft system depends greatly on how batches and pipelines are scheduled. If not enough requests are accumulated in one batch under high load, overall processing will be inefficient, leading to low throughput and high latency. On the other hand, if too many requests are accumulated in one batch, latency will be needlessly high, as early requests wait for later requests to arrive.

While we are still investigating the best policy, our goal is to minimize the average delay for
requests under dynamic workloads. Before we had implemented pipelining in LogCabin, it used a simple double-buffering technique. The leader would keep one outstanding RPC to each follower. When that RPC returned, it would send another one with any log entries that had accumulated in the meantime, and if no more entries were available, the next RPC would be sent out as soon as the next entry was appended. This approach is appealing because it dynamically adjusts to load. As soon as load increases, entries will accumulate, and the next batch will be larger, improving efficiency. Once load decreases, batches will shrink in size, lowering latency. We would like to retain this behavior for pipelining. Intuitively, in a two-level pipeline, we would like the second batch to be started halfway through the processing time for the first batch, thus halving the average delay. However, guessing when a batch is halfway done requires estimating the round-trip time; we are still investigating the best policy to use in LogCabin.

### 10.3 Preliminary performance results

We have not yet analyzed the performance of LogCabin in depth, but we have taken some initial measurements. The experimental setup is summarized in Table 10.1. In the benchmark, a single client process connects to the leader of a LogCabin cluster. Varying numbers of client threads issue operations to the replicated state machine to set a 1,024-byte value. Each client thread repeatedly issues a request on a shared TCP connection, waits for the result from the leader’s state machine, then issues its next request.

Figure 10.3(a) shows the current throughput of LogCabin. Using a multi-threaded client with 100 threads, a three-server cluster sustains about 19,500 kilobyte-sized writes per second. As expected, performance degrades when using larger clusters, since the leader has to send each entry to a larger number of followers.

Figure 10.3(b) shows the current latency of LogCabin. The latency for a kilobyte-sized write is about 0.7 ms for a single-server cluster and about 1.0 ms for two- to five-server clusters. This

<table>
<thead>
<tr>
<th>code</th>
<th>LogCabin [86], written in C++11</th>
</tr>
</thead>
<tbody>
<tr>
<td>OS</td>
<td>x86-64 RHEL6 (Linux 2.6.32)</td>
</tr>
<tr>
<td>CPU</td>
<td>Xeon X3470 (4 cores, 8 hyperthreads)</td>
</tr>
<tr>
<td>disk</td>
<td>ext4 file system on Intel DC S3500 SSDs (1 SSD per server; write caching off)</td>
</tr>
<tr>
<td>network</td>
<td>Protocol Buffers [111] over TCP/IP over 1 gigabit Ethernet</td>
</tr>
<tr>
<td>configuration</td>
<td>in-memory state machine, no log compaction</td>
</tr>
</tbody>
</table>

Table 10.1: Experimental setup.
Figure 10.3: Preliminary latency and throughput measurements of LogCabin. In each test, a single client process connects to the leader of a LogCabin cluster of varying size. In (a), varying numbers of client threads issue operations to the state machine to set a 1,024 byte value; in (b), only one client thread is used. Each client thread repeatedly issues a request on a shared TCP connection, waits for the result from the leader’s state machine, then issues its next request. Each data point represents the mean of five runs of approximately 10 seconds each; error bars show the minimum and maximum values across the five runs (the range is very small for most points).
includes the time to write one kilobyte durably to disk, which we measured in a microbenchmark to be about 0.25 ms.

The initial measurements are encouraging, and we think the current performance would be sufficient for a large class of applications. However, there is still much room for improvement. For example, gigabit Ethernet would limit the performance of a three-server cluster to about 60,000 kilobyte-sized writes per second, and LogCabin’s current throughput is only one third of that.

10.4 Conclusion

There are many performance aspects of Raft we would like to analyze in the future. Most importantly to normal operation, we would like to analyze the latency and throughput for write operations and for read-only operations under varying load. There are various performance questions that arise during exceptional circumstances that we would also like to analyze:

- How quickly do clients find the leader?
- How quickly does a new leader commit its first entry, including how quickly does a leader discover where its followers’ logs diverge?
- What is the effect of follower failures on normal operation?
- How long does it take to reconfigure the cluster, and what is its effect on normal operation?
- How long does it take to compact the log, and what is its effect on normal operation?
- How long does it take a server/cluster to restart?

Our performance goal with Raft was to match current algorithms such as Multi-Paxos, while improving understandability. Rather than wanting to build the fastest system, we wanted to enable others to build consensus-based systems that were competitive in performance. Though LogCabin is not yet well-optimized, preliminary results show that it achieves reasonable latency and throughput: writing kilobyte-sized objects to a three-server cluster takes about 1.0 ms per operation with a single client thread, and the system processes 19,500 operations per second when using 100 client threads.
Chapter 11

Related work

This chapter discusses the strengths and weaknesses of Raft in the context of related work. Section 11.1 first gives a brief introduction to other consensus algorithms and compares them to Raft at a high level. Then, Sections 11.2–11.8 focus on more specific details of how these consensus algorithms compare to Raft. Finally, Section 11.9 discusses work related to evaluating understandability.

11.1 Overview of consensus algorithms

This section introduces existing consensus algorithms that are comparable to Raft, specifically Paxos, Viewstamped Replication, and Zab. Like Raft, these algorithms handle fail-stop but not Byzantine failures, and they do not rely on time for safety (the key properties of practical consensus algorithms can be found in Section 2.1). Readers may also be interested in van Renesse et al.’s more theoretical comparison of these algorithms [109].

Other consensus algorithms exist for different system models, but these are less commonly used. Notably, some algorithms address Byzantine consensus, where arbitrary failures and misbehaviors are possible [13, 65, 76]; these are more complex and lower in performance than algorithms under the fail-stop model.

11.1.1 Paxos

Paxos (most commonly Multi-Paxos) is the most widely deployed consensus algorithm today:

Megastore and Spanner use Paxos for all of their data storage.


- The open-source Ceph storage system uses Paxos to store its cluster map, the data structure that allows clients to find where objects are located [112, 14].

- Recently, eventually-consistent data stores such as Cassandra [1] and Riak [6] have added Paxos to provide linearizable access for some data. Cassandra appears to use an unoptimized implementation of Basic Paxos [26], and a future release of Riak will include an implementation of Multi-Paxos [9].

Paxos is a broad term for a whole family of consensus protocols. Lamport’s original description of Paxos [48] presents sketches for a complete system but not in enough detail to implement. Several subsequent papers attempt to explain Paxos [49, 60, 61], but they also don’t explain their algorithms completely enough to implement. There are many other elaborations of Paxos, which fill in missing details and modify Paxos to provide a better foundation for implementation [108, 46]. Additionally, we developed our own explanation for and elaboration of Paxos in a video lecture as part of the Raft user study [88]; the Multi-Paxos variant we used is summarized in Figure A.2. Unfortunately, all of these elaborations of Paxos differ from each other. This is burdensome for readers, and it also makes comparisons difficult. Ultimately, most implementations bear little resemblance to the Paxos literature, and some may even deviate so far from Paxos as to resemble Raft. After reading an earlier draft of the Raft paper, one Spanner developer made the following remark during a talk:

Our Paxos implementation is actually closer to the Raft algorithm than to what you read in the Paxos paper. [43]

For the purpose of this chapter, we have tried to compare Raft to common ideas found in Multi-Paxos elaborations, but we did not limit our discussion to a particular algorithm.

Chapter 2 discussed how Paxos is difficult to understand and is a poor foundation for building systems. Its single-decree formulation is difficult to decompose, and Multi-Paxos leaves the log with too much nondeterminism and too little structure (e.g., it can have holes). Multi-Paxos uses only a very weak form of leadership as a performance optimization. These problems make Paxos needlessly complex, which burdens both students and systems builders.
11.1.2 Leader-based algorithms

Viewstamped Replication and Zab are two leader-based consensus algorithms that are closer in structure to Raft and therefore share many of Raft’s advantages over Paxos. As in Raft, each algorithm first elects a leader, then has that leader manage the replicated log. The algorithms differ from Raft in how they handle leader election and repairing inconsistencies in the logs after leader changes; the next sections in this chapter go into more details on these differences.

Oki and Liskov’s Viewstamped Replication is a leader-based consensus algorithm developed around the same time as Paxos. The original description [83, 82] was intertwined with a protocol for distributed transactions, which may have caused many readers to overlook its contributions. The core consensus algorithm has been separated in a recent update called Viewstamped Replication Revisited [66], and Mazières [77] also expanded on the details of the core algorithm before Liskov’s update. Though Viewstamped Replication is not widely used in practice, it was used in the Harp File System [67].

Zab [42], which stands for ZooKeeper Atomic Broadcast, is a much more recent algorithm that resembles Viewstamped Replication. It is used in the Apache ZooKeeper coordination service [38], which is the most popular open-source consensus system today. A cluster membership change mechanism was recently developed for Zab [104] and is scheduled for a future ZooKeeper release [113].

Raft has less mechanism than Viewstamped Replication and Zab because it minimizes the functionality in non-leaders. For example, we counted the message types Viewstamped Replication Revisited and Zab use for basic consensus and membership changes (excluding log compaction and client interaction, as these are nearly independent of the algorithms). Viewstamped Replication Revisited and Zab each define 10 different message types, while Raft has only 4 message types (two RPC requests and their responses). Raft’s messages are a bit more dense than the other algorithms’, but they are simpler collectively. In addition, Viewstamped Replication and Zab are described in terms of transmitting entire logs during leader changes; additional message types will be required to optimize these mechanisms so that they are practical.

Zab presents a slightly stronger guarantee than Raft for clients issuing concurrent requests. If a client pipelines multiple requests, Zab guarantees that they are committed in order (if at all); this property is called FIFO client order. For example, this allows a client to issue a bunch of changes and then release a lock, all asynchronously; other clients will see the changes reflected in the replicated state machine before they see the lock released. Paxos does not satisfy this property, since commands are assigned to log entries with few constraints; see [42]. Raft and Viewstamped
Replication could provide the same guarantee as Zab, since their leaders append new entries in order to the log. However, some extra care would be required to prevent network and client retries from reordering the client’s commands to leaders.

11.2 Leader election

This section discusses how different consensus algorithms address leader election. Raft uses an approach with very little mechanism, while other algorithms are generally more complex without offering practical advantages.

In a broad sense, leader election includes the following four issues, which the following subsections discuss in depth:

1. Detecting a failed leader.
   Raft uses heartbeats and timeouts.

2. Neutralizing deposed leaders.
   In Raft, candidates propagate a new term number while soliciting votes and replicating the log.

3. Selecting a server to be the new leader.
   Raft uses randomized timeouts, and the first candidate to time out usually becomes leader.
   Voting ensures that there is at most one leader per term.

4. Ensuring the leader has all committed entries.
   In Raft, the log comparison check during voting ensures that a new leader already has all committed entries; no log entries are transferred.

11.2.1 Detecting and neutralizing a failed leader

In all practical settings, it is impossible to distinguish a failed server from a slow server; this is the key characteristic of an asynchronous system. Fortunately, practical consensus algorithms preserve safety even if leaders are suspected of failing when they are simply slow. Thus, failure detection only needs to detect failed servers eventually (completeness) and not suspect available servers with high probability (accuracy). These weak requirements are easily satisfied in practical systems by using heartbeats and timeouts.
Various failure detectors built on heartbeats and timeouts have been discussed in the theoretical literature \cite{16}. $\Diamond P$ (or equivalently, $\Omega$) is a failure detector with nice theoretical properties: eventually (after some unknown period of time), it will be perfectly correct and accurate. It does so by increasing its timeouts every time a suspicion is incorrect; eventually, its timeouts will be so large that it makes no false suspicions. However, this behavior is impractical for real systems, which care about availability: if the timeout value grows too large, the cluster will wait too long to detect a leader failure. It is better to falsely suspect a leader of failure when it is slow than to wait around to be sure. Therefore, Raft’s timeouts are fixed low enough to satisfy the system’s availability requirements.

Paxos, Zab, and Viewstamped Replication either do not specify a failure detector or briefly mention the use of timeouts but do not spell out the details. This may be because approaches to failure detection are mostly independent of the consensus algorithm. However, we found that combining heartbeats with other messages has practical benefits. For example, Raft’s AppendEntries RPC not only serves as a heartbeat but also informs followers of the latest commit index.

Since failure detectors can mistakenly report the leader as having failed when it is in fact slow, a suspected leader must be neutralized. The various consensus algorithms handle this similarly using a monotonically increasing number (called a term in Raft, a proposal number in Paxos, a view in Viewstamped Replication, or an epoch in Zab). Once a server has seen a larger number, it will no longer accept requests from a leader with a smaller number. Most algorithms, including Raft, inform the sender that it is stale when a server receives such a request; in some descriptions of Paxos, however, the recipient does not reply.

Algorithms assign term numbers to servers in two different ways. Zab and Raft use voting to ensure there is at most one leader per term: if a server is able to collect a majority of votes, it has exclusive use of that term number for replicating log entries. Paxos and Viewstamped Replication divide the space of numbers so that servers do not compete for particular numbers (e.g., by allocating numbers to servers in a round-robin fashion). There does not seem to be a practical difference between these two approaches, since voting must occur in either case.

11.2.2 Selecting a new leader and ensuring it has all committed entries

Algorithms differ in which server they select as leader, as summarized in Table 11.1. Paxos and Zab choose any server as leader, while the other algorithms restrict which server can become leader. One advantage of Paxos and Zab’s approach is that they can accommodate preferences about which server should be leader during leader election. For example, if a deployment performs best when
Table 11.1: Summary of how different algorithms select a new leader. The “new leader” column shows which servers may become the new leader. The “vote collector” column shows which server solicits votes; in all but the original Viewstamped Replication paper, this is the candidate for leadership. The “handles preferences” column shows which algorithms are able to accommodate preferences in which server becomes leader during election; other algorithms would need separate leadership transfer mechanisms to accommodate this.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>New Leader</th>
<th>Vote Collector</th>
<th>Handles Preferences</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paxos</td>
<td>any server</td>
<td>new leader</td>
<td>yes</td>
</tr>
<tr>
<td>VR</td>
<td>has up-to-date log</td>
<td>view manager</td>
<td>no</td>
</tr>
<tr>
<td>VRR</td>
<td>determined by view number</td>
<td>new leader</td>
<td>no</td>
</tr>
<tr>
<td>Zab</td>
<td>any server</td>
<td>new leader</td>
<td>no</td>
</tr>
<tr>
<td>Raft</td>
<td>has up-to-date log</td>
<td>new leader</td>
<td>yes</td>
</tr>
</tbody>
</table>

a server from a particular datacenter acts as leader, Paxos or Zab can allow that server to become leader. The other algorithms are not able to do so because they constrain which server may become leader; they need a separate leadership transfer mechanism (as described in Chapter 3 for Raft) to accommodate such preferences.

Viewstamped Replication Revisited uses a different round-robin approach for choosing which server becomes leader. The leader is a function of the view (term) number: in an $n$-server cluster, a server $i$ is the leader for view $v$ if $v \% n = i$. This approach has the advantage that clients can likely guess and find the leader based on the current view number (to do this, clients must track the current configuration and view number). However, it may result in additional delays if the designated leader for a view is unavailable or if servers have different notions of the current view.

The original Viewstamped Replication algorithm is closest to Raft in that only a server whose log is as up-to-date as a majority of the cluster can become leader. This has a big advantage in that it avoids transferring log entries to the new leader; it simplifies the flow of data to go only from clients to leaders to followers. Viewstamped Replication uses one server to manage the election process (the view manager) and a different server becomes the leader. The view manager chooses the server with the most up-to-date log of a majority of the cluster to be the new leader, then informs that server of its new leadership role. In Raft, the same server both runs the election and becomes leader, which avoids some mechanism and reduces state space complexity. Zab also suggests choosing the new leader as having a sufficiently up-to-date log (like Raft) as a possible optimization, and this optimization is apparently implemented in ZooKeeper [94].

Paxos, Viewstamped Replication Revisited, and (unoptimized) Zab need additional mechanism to ensure the new leader has all committed entries, since they do not choose the leader based on
its log. In Paxos, the leader typically runs both phases of single-decree Paxos for each log entry in which it does not know the committed value, until it reaches a log index for which no available server has seen any more proposals. This may result in significant delays until the new leader catches up. Viewstamped Replication Revisited and Zab are described as if servers send their entire logs to the new leader and the new leader adopts the most up-to-date one. This is a nice model but is impractical for large logs; both papers suggest optimizing this by sending fewer entries but do not spell out the details.

11.3 Log replication and commitment

All consensus algorithms specify how to send new log entries to other servers and when to mark them committed. This is usually done in one round of communication from the leader in the normal case, and it is usually straightforward to apply batching and pipelining to make replicating multiple entries faster.

The algorithms differ in how far they can proceed out of order. Raft, Zab, and Viewstamped Replication must all append and commit entries to the log in order, so that followers’ logs always remain consistent with the leader’s. Traditionally, Multi-Paxos allows servers to accept and commit values for entries in any order. This does not offer Paxos a significant performance advantage, however, since commands must still be applied to the state machines in order. Raft and the other algorithms that maintain a log in order can also transmit log entries out of order; they just cannot be appended to the log this way. (In these algorithms, servers could buffer the entries outside the log until they are ready to be appended, if desired.)

The algorithms also differ in what new leaders do with existing entries in their logs, as illustrated in Figure 11.1:

- In Paxos, a new leader goes through the two phases of single-decree Paxos for each uncommitted entry it finds, rewriting and renumbering them all with its current proposal number. This either commits the local value or discovers an existing committed value. Meanwhile, it can replicate and commit but not yet apply client commands in further log slots.

- In Viewstamped Replication Revisited and Zab, a new leader transfers its entire initial log to each follower before starting its term, and the entire log is effectively renumbered with the new view. This is impractical for large logs and should be optimized to send fewer entries in practice, but the details have not been published. It is fairly easy to determine which entries
Figure 11.1: Example of how algorithms differ in which entries a new leader replicates from its log. In Paxos, the new leader for term 4 executes phases 1 and 2 of Paxos for entries 4–8 using its new proposal number, since it does not believe that those are committed. As described in the Viewstamped Replication and Zab papers, the new leader replicates its entire log to the follower. In Raft, the leader only transmits entries 5–8 to the follower, the minimal number of entries required.

- A new leader in Raft transfers just the minimal number of entries to make other servers’ logs match its own. After some back-and-forth with heartbeats to discover where the logs diverge, the only entries that are transferred are those that differ. Key to this feature is that entries are not renumbered, so the same entry will have the same index and term across logs for all time. Without this property, some servers would have an entry under its original term number, and others would have it under new term numbers. A subsequent leader would have to needlessly overwrite some of these copies, since it wouldn’t know which ones contain the same command.

By transferring log entries rather than logs, Raft allows more intermediate states than VR and Zab. These intermediate states are ambiguous in Raft, thus cannot be used for commitment (see Figure 3.7). This has three consequences.

First, if we could somehow observe a snapshot of an entire cluster, an entry in Raft can be present on a majority of servers but not committed. Instead, to determine whether an entry is committed, one must ask if future leaders must have the entry: does every server that could be elected leader with its current log have the entry in its log? If so, the entry is committed; otherwise, it is not. This requires more complex reasoning for an omniscient observer than in other algorithms: rather than counting
how many replicas of the entry exists, one must essentially execute the consensus algorithm.

Second, during operation, Raft has a two-part commitment rule, in which entries from prior terms are not directly marked committed; they are only marked committed once an entry from the current term has reached a majority of the cluster (at this point, any ambiguity is resolved). This does not significantly burden implementations, which only need a single additional if statement. Interestingly, this commitment rule would not be possible in a single-decree consensus formulation; it relies on the log formulation so that later entries can commit earlier ones.

Finally, infinite leader changes can require infinite space in Raft. Specifically, a leader has to create an entry in order to commit previous entries in order to compact them, but if it crashes first, its log will then contain an additional entry. In theory, this process could repeat and exhaust storage capacity. However, we don’t believe this to be a significant practical concern, since it would be unlikely for leader election to succeed so frequently yet leaders to fail so frequently.

An alternative to Raft’s commitment approach would be to add an extra term to logs, similar to Viewstamped Replication Revisited. The log’s term would be the term of the latest leader to replicate an entry to the log. The log’s term would usually be the same as the term of the last entry in the log, but it would be ahead briefly while new leaders catch followers up to match the leader’s initial log. If the log’s term was used during elections instead of the term of the last entry, then the commitment rule could be simplified: commitment would require a majority of servers to have the entry and the same log term. Based on its similarity to Viewstamped Replication, we think this approach would work, though we haven’t proved it correct. The downside is that this results in three terms to juggle: the server’s current term, the log’s term, and the terms in the individual entries. We think delaying commitment until the ambiguity is resolved is easier.

11.4 Cluster membership changes

Several different approaches for cluster membership changes have been proposed or implemented in other work. Most of these implement arbitrary cluster membership changes, while Raft restricts changes to single-server additions and removals. We do not know of prior work that discusses restricting changes to single-server additions and removals for simplicity, though we think it is likely that prior systems have implemented this. The remainder of this section compares related work to Raft’s joint consensus approach to arbitrary cluster membership changes, as presented in Section 4.3.
In order to ensure safety across arbitrary configuration changes, the changes must use a two-phase approach. There are a variety of ways to implement the two phases. For example, some systems (e.g., [66]) use the first phase to disable the old configuration so it cannot process client requests; then the second phase enables the new configuration. In the approach to arbitrary configuration changes in Raft, the cluster first switches to a transitional configuration called joint consensus; once the joint consensus has been committed, the system then transitions to the new configuration.

11.4.1 \( \alpha \)-based approaches

Lamport [48, 49] proposed for Paxos that the \( i^{th} \) log entry would determine the cluster membership for the \( i + \alpha^{th} \) log entry. The two phases in this approach are:

1. The new configuration is agreed upon at log entry \( i \); then
2. The new configuration takes effect at log entry \( i + \alpha \).

A cluster is able to process requests during configuration changes, up to the \( \alpha \) limit.

Unfortunately, \( \alpha \) also limits the degree of concurrency of a Paxos cluster during normal operations. If entry \( i \) is the first entry not yet known to be committed, it is possible that \( i \) could eventually end up changing the configuration; thus, servers cannot send proposals for entry \( i + \alpha \) or beyond until they learn of \( i \)’s commitment. \( \alpha \) can be configured to be large to allow for sufficient pipelining/batching of entries during normal operation, but then configuration changes take longer to take effect. To mitigate this, a server can propose no-op entries in the intervening \( \alpha - 1 \) log entries.

While Lamport’s proposal handles safety concerns quite simply, it leaves many liveness and availability questions unanswered. For example, the new servers need to learn all the decisions from the old cluster so their state machines can advance. How do the new servers get these entries, and how do the old servers know when they can shut down? How do the new servers even know what the old or new configurations are?

SMART [69] is an attempt to address these questions. In SMART, each physical server hosts one or more virtual servers, and each virtual server participates in a single cluster with a static configuration. SMART uses an \( \alpha \)-like approach for determining when one configuration should finish accepting client requests and terminate. When the old configuration receives a membership change request, it informs the new configuration to begin at a particular log index (\( \alpha \) entries later). Once the final log from the old configuration has been transmitted to a majority of the servers in the new configuration, the new configuration may start servicing client requests, and the virtual servers in the old configuration may shut down.
SMART’s model for membership changes may be challenging to implement efficiently. During the change, if one physical server is part of both the old and the new cluster, it must simultaneously run two virtual servers, one for each configuration. To make this space-efficient, some of the server’s state is moved to a separate execution model which is shared by all virtual servers on a single physical server. Unfortunately, this adds significant mechanism and complexity for implementations. In contrast, in Raft, each server participates in only one configuration at a time; it always uses the latest configuration in its log.

The \( \alpha \) and SMART approaches are incompatible with Raft’s commitment rule. In Raft, if a leader cannot append to its log, it may not be able to mark existing entries as committed. For example, suppose a leader reached its limit of \( \alpha \) uncommitted entries, then restarted and became leader again. Due to the \( \alpha \) limitation, the leader could not create any new entries in its current term, so it wouldn’t be able to mark any existing entries committed. In this case, \( \alpha \) and Raft are in conflict: \( \alpha \) requires commitment to append new entries, but Raft requires appending new entries for commitment.

If Raft’s commitment rule were not an issue (e.g., if Viewstamped Replication’s commitment rule were used instead), the \( \alpha \) or SMART approaches to membership changes could work, but Raft’s leader-based approach poses additional challenges. Log entries in Raft are only sent from the leader to other servers, so the old cluster’s leader needs to replicate all of its entries to the new cluster (as well as committing them to the old cluster). Thus, the old cluster leader would need to add the new cluster servers as non-voting members of its configuration. With the \( \alpha \) approach (but not with SMART), the need to maintain a leader in the old cluster results in scenarios where there are two leaders in a single Raft cluster: the leader of the old cluster replicates the log up to \( i + \alpha \) and cannot write beyond that, and the leader of the new cluster knows the entries up to \( i + \alpha \) are committed and replicates new log entries past \( i + \alpha \). Even if the two leaders don’t conflict over log entries, they are likely to introduce availability issues without additional mechanisms. The SMART approach is conceptually simpler for allowing multiple concurrent leaders, since the leaders are members of distinct clusters.

11.4.2 Changing membership during leader election

The original Viewstamped Replication algorithm did not include membership changes, but Viewstamped Replication Revisited and a paper by Mazières [77] each extends the original algorithm to support membership changes. Both approaches change the membership between views while the cluster has no leader. Thus, neither can process client requests during membership changes, and
neither approach is compatible with Raft, which requires a leader to transfer entries to the new 
servers.

In Viewstamped Replication Revisited, the new configuration is committed as a special log entry 
under the old configuration, then a view change (leader election) is initiated. The servers in the new 
configuration must update themselves from the old cluster before they can begin participating in the 
new view (term). Meanwhile, they cannot process client requests. When the leader and enough other 
servers have updated themselves, the cluster resumes processing client requests. The two phases in 
this approach are:

1. The old servers move to a new view, thereby stopping client requests; and
2. Once the new servers have gotten the necessary log entries, they resume processing client 
requests.

Mazières presents another approach in which the cluster membership is decided as part of view 
changes [77]. To form a new view, the cluster reaches agreement both on who the leader will be and 
on who the cluster members will be. The two phases in this approach are:

1. When a server accepts an invitation for a new view, it stops accepting requests from the leader 
of the old view; and
2. Once the server learns the view change has been agreed upon, it begins accepting requests 
from the leader of the new view.

In some cases when intervening view changes have failed, the servers must sometimes require a 
majority of the old cluster and a majority of the new cluster for agreement to begin operating in the 
new view; this is similar to joint consensus but is only used in special cases.

Mazières’s approach operates using additional messages rather than log entries, since there is no 
leader during the view change to commit log entries. This requires additional mechanism to agree 
upon and transmit configurations, which Raft avoids. Raft’s algorithm also has the advantage that 
normal requests can proceed during membership changes; in contrast, both Viewstamped Replication 
Revisited and Mazières’s approaches must temporarily stop all normal processing during 
membership changes.

Neither Viewstamped Replication Revisited nor Mazières’s approach works for Raft because 
Raft has no separate mechanism for “state transfer”. In Raft, the old servers must maintain a leader 
long enough to replicate and commit log entries to the new servers, but the Viewstamped Replication
Revisited and Mazières approaches require the cluster to be able to replicate log entries without a leader.

### 11.4.3 Zab

Zab’s approach to membership changes is the closest to Raft’s joint consensus approach, and the basic idea would also work for Raft. The two phases in Zab’s approach are:

1. A log entry containing the new configuration is committed to both a majority of the old cluster and a majority of the new cluster. The old leader may continue to replicate entries past the new configuration entry, but it may not mark any further entries committed (unless it is also part of the new cluster).

2. Then, the leader of the old cluster sends *Activate* messages to the new cluster, informing the new cluster of the configuration entry’s commitment. This enables the new cluster to elect a leader and continue operations. If the old leader is also part of the new cluster, it can continue as leader.

Raft’s joint consensus approach records state during membership changes more explicitly in the log: it uses a second log entry to activate the new configuration, whereas Zab uses *Activate* messages that are not logged. This makes Zab’s transitions and failure recovery more complex, as a server’s current configuration depends on both its log and its latest committed configuration. In Raft, on the other hand, a server always uses the latest configuration in its log, and failures are handled with no additional mechanism.

Neither algorithm stalls client operations when the old leader is also part of the new cluster, as this server continues as leader throughout and beyond the membership change. However, Zab’s treatment of leaders that are being removed from the cluster differs from Raft’s in two ways:

1. In Raft a leader that is being removed continues to commit log entries until it steps down. In Zab, however, a leader that is being removed may not commit any log entries that come after the configuration change entry in its log. It may still replicate those entries, though, and the effect of this restriction is probably small.

2. In Zab if the leader removes itself from the cluster, the new servers will begin leader election immediately, and the old leader can designate a new server to become leader immediately. In Raft the new servers wait for an election timeout, but using Raft’s leadership transfer extension (Chapter 3) can similarly avoid this delay.
ZooKeeper allows reads to be served by any server, and, without additional mechanism, clients may end up imbalanced across servers after membership changes. For example, servers that have recently been added to a cluster will have a disproportionately low number of clients connected to them. The paper describes a probabilistic algorithm to rebalance client load to the new servers after a membership change, which would also be useful for Raft implementations that allow reads from any server.

### 11.5 Log compaction

Log compaction is a necessary component of any consensus-based system, but unfortunately, the topic is neglected in many papers. We can think of two reasons why this might be the case:

1. Most of the issues of log compaction are equally applicable to all consensus algorithms. All algorithms must eventually commit each log entry, and committed entries can then be compacted without affecting the consensus algorithm much (since consensus has already been reached). Thus, from a theoretical point of view, compaction is nearly orthogonal to the consensus algorithm and may not logically belong in a paper about a consensus algorithm.

2. Log compaction involves a large number of design choices, and some of these may vary by implementation. Different approaches trade off complexity, performance, and resource utilization in different ways, and implementations may vary significantly in their requirements (for example, ranging from very small to very large state machines). Some authors attempt to describe algorithms in the most general terms possible, and it is difficult to be inclusive of all possible implementations when facing such a large design space.

This dissertation discussed several forms of log compaction. The biggest design choice is between incremental approaches (described in Section 5.3), and snapshotting, which is simpler but less efficient. Many consensus-based systems use some form of snapshotting. Raft’s snapshotting approach is very similar to that of Chubby [15], and a similar snapshotting approach is outlined briefly in Viewstamped Replication Revisited [66].

ZooKeeper [38] uses *fuzzy snapshots*: rather than taking a consistent snapshot using copy-on-write techniques, a snapshot in ZooKeeper can partially reflect later changes, thereby not representing the state of the system at a particular point in time. The changes that may or may not have already been applied to the snapshot are reapplied on server startup, resulting in a consistent state.
CHAPTER 11. RELATED WORK

(a) Traditional replicated state machine approach.

(b) Primary copy approach.

Figure 11.2: In the primary copy architecture, the primary’s state machine processes requests from clients and calculates resulting states, which its consensus module replicates into the servers’ logs. The figure shows a client submitting a request to increment a variable \( y \), which the primary translates into an operation to set \( y \) to 2.

Unfortunately, fuzzy snapshots are covered by a US patent [95], and they are also more difficult to reason about than consistent snapshots.

11.6 Replicated state machines vs. primary copy approach

The original Viewstamped Replication paper and ZooKeeper operate slightly differently from traditional replicated state machines, using a primary copy architecture instead. The primary copy architecture is illustrated in Figure 11.2. It is similar to replicated state machines in that each server still has a consensus module, a state machine, and a log. However, the primary’s (leader’s) state machine processes requests as soon as they arrive from clients, instead of waiting for them to be committed. It then computes the state resulting from each request, and the final state, rather than the original requests, is replicated in the log using consensus. Once the log entries are committed, the effects of the client requests are externalized to clients. (For linearizability, the primary should also include client responses in the log entries, allowing backups servers to return the same response in case clients retry; see Chapter 6.)

From the point of view of the consensus algorithm, the primary copy approach is very similar to
replicated state machines. Thus, nearly all of the Raft algorithm applies equally well to the primary copy approach. However, the state machine and overall system are somewhat more complex in the primary copy approach. They differ in three ways.

First, the primary’s state machine in primary copy systems reflects uncommitted entries in the log, whereas in replicated state machines, the state machines only reflect committed entries. This distinction is necessary for primaries to produce the resulting states when they receive client requests, but it introduces two complications: the state machine must take caution not to externalize any uncommitted state, and if another server becomes the primary, the old primary’s state machine needs to roll back its recent uncommitted changes.

Second, the log in the replicated state machine approach includes all client requests, even those that ended up having no effect. For example, a conditional write operation whose condition was not met would still occupy space in the log. In the primary copy approach, the primary would not need to append anything new to its log for such failed operations (it would only need to wait until it was safe to externalize the response). On the other hand, this is unlikely to have a significant effect on the system’s capacity, as logs must eventually be compacted in either approach.

Third, the state machines in the replicated state machine approach must be deterministic, since every server must arrive at the same result after applying the same series of client requests. For example, the effects of client requests must not depend on each server’s current time. In the primary copy approach, however, the primary’s state machine need not be deterministic; it may do anything it likes with the request, as long as the state change it produces is deterministic. Fortunately, a hybrid approach allows replicated state machines to overcome this limitation in most cases: the server receiving a client request can augment that request with additional nondeterministic inputs, such as its current time and a random number, before appending the request into the replicated logs. All of the servers’ state machines can then process the augmented request deterministically.

11.7 Performance

Many papers have proposed performance enhancements to Paxos and other consensus algorithms. Although these performance enhancements can be useful, implementers will have to judge which, if any, are appropriate in their situations. Prior to describing the enhancements in related work, we discuss several considerations that may be significant in these decisions.

First, others before us have recognized that performance of consensus is sometimes secondary to understandability or ease of implementation. For example, Boxwood [72] uses an implementation
of Paxos that only processes one log entry at time (in sequence with no batching or pipelining). The authors note:

This makes the implementation slightly easier without sacrificing the effectiveness of the protocol for our purposes.

It would be unwise to use a more complex algorithm or implementation for performance reasons if no application will ultimately reap the benefits.

Second, the performance of a single consensus group is fundamentally limited, since each operation must involve more than half of the servers in the cluster. The best case throughput for a consensus group cannot exceed twice that of a single server, since each server needs to process a majority of commands. The only way to scale consensus to large clusters is to use more independent consensus groups (see Chapter 2) and to minimize synchronization across groups.

Possible latency improvements are also limited, especially for datacenter networks. The best case for latency is replicating directly from the client to the majority of the cluster nearest the client, whereas in leader-based algorithms, the client replicates to the leader, then the leader replicates to the nearest half of the other servers in the cluster. The possible improvement thus depends on the geographical layout of the client and servers; the worst case latency can improve from circling the globe twice per request to circling it just once.

Third, several important performance gains can be achieved without fundamentally changing the algorithm:

- Most practical implementations of consensus employ some form of pipelining and/or batching of log entries. Chapter 10 discussed batching and pipelining in Raft, and Santos and Schiper [100] analyzed trading off batching and pipelining in the context of Paxos. Unfortunately, they suggest optimizing for throughput at the expense of latency, and their model does not include writes to stable storage.

- The leader’s outbound network usage, which is typically the limiting factor in throughput for leader-based algorithms, can also be reduced without fundamentally changing the algorithm. For example, the leader can use chain replication [30, 110] (in which the leader replicates to the first follower, which in turn replicates to the second, etc.) or network multicast to replicate entries to its followers; all followers receive copies of the log entries, but the leader only has to transmit each log entry once. Alternatively, the followers can replicate batches of commands into each other’s memory, and the leader can then order these batches into the replicated log without transmitting the full command data; S-Paxos [8] fleshed out the details for Paxos.
Finally, many of the performance optimizations in this section have unfortunately been patented in the US. We have tried to warn readers of patents that we are aware of, and we sincerely hope that software patents will be reformed or abolished in the US soon (the Electronic Frontier Foundation describes why [25]).

11.7.1 Reducing leader bottleneck

Many optimizations focus on reducing the leader as a performance bottleneck. As a single server, the leader has limited resources and may be located inconveniently in wide-area deployments. Thus, optimizations have the potential to:

- Increase throughput by using network links in a more balanced way;
- Decrease latency by avoiding the (possibly long) network hop to involve the leader; and
- More evenly balance load between the servers.

Unfortunately, most of these optimizations are in conflict with Raft’s strong leader approach. Raft leverages its strong leader for understandability and reducing mechanism, and this key design choice is at odds with reducing the leader’s involvement in normal operations. Thus, if Raft were modified to support these optimizations, the end result would differ considerably from the Raft algorithm, and it would probably be significantly harder to understand.

Rotating leader (Mencius)

In a US patent [56], Lamport et al. describe an idea to divide a replicated log such that different servers act as leader for different log indexes. For example, leadership can be assigned round-robin to all the servers in the cluster. Mencius [74] applies this idea to Paxos and works out many of the details needed for a practical implementation. For example, servers in Mencius can efficiently skip their turns if they have no client requests to propose.

Mencius can improve the cluster’s throughput since all servers can propose requests. It can also improve latency when servers are separated by wide-area links, as clients can submit their requests to a nearby server. However, its design also has two potential downsides for performance:

1. A slow server can delay state machines from applying further log entries, since it needs to propose a value or skip its turn (or worse, another server must revoke its turn) in order to make progress. This impacts latency but not throughput.

2. Similarly, any failed server can result in reduced performance until the cluster revokes its assigned log entries. In contrast, a non-leader failure in Multi-Paxos does not usually affect
performance.

We think Raft could be extended to support Mencius-like operation. However, it would add so much complexity to Raft that the end result might hardly resemble Raft at all.

**Offloading leadership burden to clients (Fast Paxos)**

Fast Paxos [52] (covered by a US patent [53]) describes an approach to reducing the leader bottleneck in which clients propose commands directly to the cluster servers, rather than submitting them to the leader to propose. This is advantageous for latency when the client is located far from the leader and the leader is far from the other servers. It also eliminates a network hop, which can improve latency even if all the servers are located in a single datacenter.

To allow clients to propose requests directly, a leader first executes the first phase of Paxos on the cluster, resolves any proposed but uncommitted log entries, and tells clients of its proposal number. Then, using the leader’s proposal number, a client can directly propose a command to all servers in the cluster. The client does not specify a log index with its command; instead, each server assigns the command to its first unused log entry. If a single client proposes a command at a particular time, the servers will typically agree on the log index for the command. If the client gets a fast quorum of the servers to accept the command for the same log index, it is committed; a fast quorum typically requires \( \lceil \frac{3N}{4} \rceil \) servers.

Practically speaking, a client often needs to learn the state machine’s output as a result of its command execution; it is not always enough to know that the command is committed. This requires not the client but some server to learn that the command was committed. Servers can send each other their accept responses, along with the command that they accepted, to determine whether commitment indeed occurred. Once a server learns that a command was committed, it can apply the command (in log order), and return the result of its state machine to the client.

If two clients propose distinct values simultaneously, the command may not commit using a fast quorum. Recovering from this situation can either be coordinated by the leader or uncoordinated. In coordinated recovery, the leader selects one of the accepted values and initiates the second phase of Paxos using a slow quorum, which typically requires \( \lceil N/2 \rceil + 1 \) servers. In uncoordinated recovery, the servers independently try to choose the same value, and they try again using a fast quorum.

Fast Paxos can help reduce latency under low load, but if clients frequently conflict, any performance improvements may be negated by the cost of recovery. Moreover, Fast Paxos is fairly complex in its messaging pattern and use of two types of quorums, and it may not be desirable to
move so much processing to the client. It might be possible to make Raft work similarly to Fast Paxos; again, however, the end result would probably not resemble Raft very much.

**Exploiting commutativity (Generalized and Egalitarian Paxos)**

Generalized Paxos [51] and Egalitarian Paxos [80] both exploit commutativity (non-interference) in state machine commands. The intuition is that, if commands A and B commute, then one state machine can apply A then B, and another can apply B then A, and they will still arrive at the same resulting states. To support this, state machines must identify which operations commute, and the consensus algorithm uses this information to determine when conflicts occur. When conflicts do not occur, the processing is quite efficient, but if commands that are proposed concurrently do not commute with each other, the algorithms require an additional round of communication.

Generalized Paxos [51] (covered by a US patent [56]) extends Fast Paxos to avoid recovery when operations commute. It is able to achieve the fast path performance of Fast Paxos even when multiple clients are proposing commands, as long as those commands do not interfere.

Egalitarian Paxos [80] has clients send their commands to the nearest server, then any server can commit a command with just one round of communication as long as other commands that are proposed concurrently commute with it. It has a smaller fast quorum than Generalized Paxos by one server.

Both Generalized and Egalitarian Paxos balance load well between servers, since no leader is needed to commit commands when operations do not interfere. They are also able to achieve lower latency than Raft in WAN settings, since they do not need to include the leader (they can involve only the closest servers to the client). However, both of these protocols add significant complexity to Paxos.

**11.7.2 Reducing number of servers (witnesses)**

There are several ways to reduce the number of servers involved in most operations without losing fault tolerance; these are summarized in Table 11.2. The first, which works with all consensus algorithms, is to simply replicate entries to a bare majority rather than the full cluster (called “thrifty” in [80]). This halves the network load for the leader during normal operation, since it only has to replicate entries to half the cluster (it can replicate the entries to the others during idle periods). However, this optimization can result in delays when servers fail, as servers that will need to become part of the quorum might have fallen far behind. This impacts Paxos the least, since the new
replication | servers | state machines | delay when server fails
--- | --- | --- | ---
Traditional consensus | 5 | 5 | no delay
Thrifty | 5 | 3 | no delay for out-of-order logs; replicate missing entries for in-order logs
Harp/Cheap Paxos | 3 + 2 | 3 | no delay
Primary-backup | 3 | 3 | communicate with external system to remove failed server

**Table 11.2:** Summary of approaches to reducing the number of servers involved in each consensus decision. In the sample configurations shown, each approach can tolerate two server failures with no possibility of data loss. The “servers” column shows the number of servers required; the Harp/Cheap Paxos approaches need three fully capable servers and two additional smaller servers. The “state machines” column shows the number of state machines that are nearly up-to-date with the replicated state machine; these can be useful to service client requests. The “delay when server fails” column describes delays that may arise when a single server fails.

server can accept later entries before accepting earlier ones; it impacts Viewstamped Replication, Zab, and Raft more, since the new server’s log has to be brought entirely up-to-date before it can accept new entries.

Harp [67] extends Viewstamped Replication to take this idea one step further: witnesses are servers that only participate in voting but do not normally participate in log replication and do not have state machines at all. When a server fails, a witness steps in to store log entries for that server until it recovers or is replaced. Witnesses allow the cluster to make consensus decisions even when some of the main servers have failed. As the resource requirements for witnesses are lower than for normal servers, they can run on limited hardware or as a secondary process on other servers. We think Raft could also support witnesses in a similar way. Cheap Paxos [57] (covered by a US patent [58]) is similar to Harp, but claims to require even less powerful servers as witnesses.

Trading off recovery time even more, a primary-backup replication scheme removes a minority of the cluster altogether (this is orthogonal to the replicated state machine vs. primary-copy distinction discussed in Section 11.6). This approach is used in Apache Kafka [93]. The primary replicates log entries to all of the backups and waits for all the backups to acknowledge each entry before committing it. If the primary fails, any of the backups’ logs is equally suitable to become the new primary, but the old primary needs to be excluded from the cluster in case it returns. If a backup fails, it too needs to be excluded from becoming an eligible primary in the future. The group can rely on an external consensus service to select a new primary and exclude servers from becoming primary. To restore its original replication factor after a failure, the primary can catch a new server up, then...
mark it in the external consensus service as eligible to become primary. In an $n$-server cluster, this approach can recover from $n - 1$ failures (with the help of an external consensus service during recovery), and it only needs to send $n - 1$ messages to replicate each log entry. However, it may take longer to recover from failures, and similarly it is not able to mask stragglers (slow servers) as well as consensus does.

11.7.3 Avoiding persistent storage writes

Many papers suggest using replication rather than stable storage for durability. For example, in Viewstamped Replication Revisited, servers do not write log entries to stable storage. When a server restarts, its log is not used for voting until it learns the current information (its disk is only used as an optimization to avoid network transfers). The trade-off is that data loss is possible in catastrophic events. For example, if a majority of the cluster were to restart simultaneously, the cluster would have potentially lost entries and would not be able to form a new view. Raft could be extended in similar ways to support disk-less operation, but we think the risk of availability or data loss usually outweighs the benefits.

11.8 Correctness

The consensus community has primarily focused its correctness efforts on proofs of safety. Most of the widely accepted consensus algorithms have been proven safe in some form, including single-decree Paxos [48, 91, 55], Multi-Paxos [10, 101], EPaxos [79], and Zab [41]. We have only found informal sketches for Viewstamped Replication [66].

There are various approaches to proofs. On one axis, proofs range from less formal to more formal. Informal sketches are useful for building intuition but might overlook errors. For Raft, we have developed a fairly detailed (semi-formal) proof and have also included informal sketches for intuition. The most formal proofs are machine-checked; they are so precise that a computer program can verify their correctness. These proofs are not always easy to understand, but they establish the truth of the statements proven with complete certainty. Machine-checked proofs are not yet standard in distributed systems (they are more popular in, for example, the programming languages community), and we struggled to create one ourselves. However, recent work argues for this approach [54, 101], and the EventML [101] authors have shown their approach can be feasible for consensus by proving Multi-Paxos correct. Pairing machine-checked proofs with informal sketches can get the best of both worlds, and we hope to see the distributed systems community move in that
direction.

Proofs also range in how directly they apply to real-world systems. Some prove properties on very simplified models; these can aid understanding but have limited direct value for the correctness of complete systems. For example, real systems vary so much from single-decree Paxos that they may not benefit much from its proofs. Other proofs operate on more complete specifications (e.g., the Raft proof presented in Appendix B and the proof for EPaxos [79]); real-world implementations are closer to these specifications, so these proofs are closer to proving properties on real-world code. Some proof systems can even generate working implementations, which eliminates the possibility of errors in translation from the specification to the implementation (e.g., EventML [101]). However, this approach has not been very popular in practice so far, perhaps because real-world systems have additional needs, such as performance, that are harder to accommodate in the generated code.

We have not found many proofs of liveness or availability (nor have we contributed any for Raft). These properties may be harder to formalize, but we hope to see a greater emphasis on this in the future.

11.9 Understandability

Studies involving human factors are common in other areas of computer science, namely Human-Computer Interaction (HCI). HCI researchers typically iterate on designs using empirical measurements, using incremental results from the study to guide improvements to their designs. To make this possible, the study must be relatively easy to repeat and relatively low in cost. A typical HCI study asks participants to learn and perform a task using a user interface, which takes little preparation and may only require a few minutes per participant. In contrast, our primary goal was to compare Raft and Paxos, not to iterate on Raft, and the cost of the Raft study made it difficult to apply an iterative approach (we needed to prepare teaching materials and quizzes, and each participant needed to invest several hours in the study). Now that we have shown that Raft is easier to understand than Paxos, it may be feasible to do further iterative studies (A/B testing) to find better variations of Raft or better variations of its explanation.

Side-stepping human factors altogether, NetComplex [18] proposed a “metric to quantify the notion of algorithmic complexity in network system design”. The metric calculates the distributed dependencies of state, where the complexity of each state variable is the sum of the complexity of its dependencies. The paper also compares the complexity of two-phase commit and single-decree Paxos according to this metric; as expected, it finds Paxos to be more complex.
Clearly a formula for quantifying the complexity or understandability of an algorithm would be very useful. However, we do not know whether the formula proposed in the NetComplex paper is the right one. Many factors contribute to complexity, and their relative importance and the interactions between them are not well understood. It is also not obvious how to apply the proposed formula to the complete Raft algorithm, which is much larger than the examples given in the paper (but we would be very interested in seeing the result).
Chapter 12

Conclusion

Our goal with this dissertation was to create a better foundation for learning consensus and building replicated state machines. When we set out to learn consensus ourselves, we found the time and effort required to understand existing algorithms was too high, and we worried that this burden might be prohibitive for many students and practitioners. We were also left with significant design work before we could build a complete and practical system using consensus. Thus, we designed Raft as a more understandable and practical algorithm to serve as a better foundation for both learning and systems building.

Several aspects of Raft’s design contribute to its understandability. At a high level, the algorithm is decomposed differently from Paxos: it first elects a leader, then the leader manages the replicated log. This decomposition allows reasoning about Raft’s different subproblems (leader election, log replication, and safety) relatively independently, and having a strong leader helps minimize state space complexity, as conflicts can only arise when leadership changes. Raft’s leader election involves very little mechanism, relying on randomized timeouts to avoid and resolve contention. A single round of RPCs produces a leader in the common case, and the voting rules guarantee that the leader already has all committed entries in its log, allowing it to proceed directly with log replication. Raft’s log replication is also compact and simple to reason about, since it restricts the way logs change over time and how they differ from each other.

Raft is well-suited for practical systems: it is described in enough detail to implement without further refinement, it solves all the major problems in a complete system, and it is efficient. Raft adopts a different architecture that is more applicable for building systems: consensus is often defined as agreement on a single value, but in Raft we defined it in terms of a replicated log, as this is needed to build a replicated state machine. Raft manages the replicated log efficiently by leveraging
its leader; committing a request requires just one round of RPCs from the leader. Moreover, this dissertation has mapped out the design space for all the major challenges in building a complete system:

- Raft allows changing the cluster membership by adding or removing a single server at a time. These operations preserve safety simply, since at least one server overlaps any majority during the change. More complex changes in membership are implemented as a series of single-server changes. Raft allows the cluster to continue operating normally during changes.

- Raft supports several ways to compact the log, including both snapshotting and incremental approaches. Servers compact the committed portions of their logs independently; the main idea involves transferring responsibilities for the start of the log from Raft itself to the server’s state machine.

- Client interaction is essential for the overall system to work correctly. Raft provides linearizability for its client operations, and read-only requests can bypass the replicated log for performance while still providing the same consistency guarantees.

This dissertation analyzed and evaluated various aspects of Raft, including understandability, correctness, and the performance of leader election and log replication. The user study showed that, after students learned Raft or Paxos, 33 of 43 of them were able to answer questions about Raft better, and 33 of 41 stated they thought Raft would be easier to implement or explain than Paxos. The proof of safety helps establish Raft’s correctness, and the formal specification is useful for practitioners, as it eliminates any ambiguities in Raft’s description. The randomized leader election algorithm was shown to work well in a variety of scenarios, typically elected a leader in less than one second. Finally, measurements showed that the current version of LogCabin can sustain about 20,000 kilobyte-sized writes per second with a three-server cluster.

We are encouraged by Raft’s fast adoption in industry, which we believe stems from its understandability and its practicality. One person’s dilemma highlights both the problems that Raft set out to solve and the benefits that it offers. Nate Hardt built a Paxos-based system at Scale Computing and had been struggling over the past year to iron out the issues with his implementation. He is now close to having an efficient, working system, but after discovering Raft, he is considering rebuilding the system with Raft. He believes his team would be able to more readily help with a Raft implementation, since they can understand the algorithm more easily and learn about all of the aspects of a complete system. Fortunately, others starting on new consensus projects have an easier choice.
Many have already been inspired to build Raft systems just for the pleasure of learning, speaking to its understandability; others are implementing Raft for production use, speaking to its practicality.

12.1 Lessons learned

I have learned many things during my years in graduate school, from how to build production-quality systems to how to conduct research. In this section I briefly describe some of the important lessons that I can pass on to other researchers and systems-builders.

12.1.1 On complexity

John once told me I had a “high tolerance for complexity.” At first I thought that was a compliment, that I could handle things that lesser humans could not. Then I realized it was a criticism. Though my ideas and code solved the problems they were meant to address, they introduced an entirely new set of problems: they would be difficult to explain, learn, maintain, and extend.

With Raft, we were intentionally intolerant of complexity and put that to good use. We set out to address the inherently complex problem of distributed consensus with the most understandable possible solution. Although this required managing a large amount of complexity, it worked towards minimizing that complexity for others.

Every system has a complexity budget: the system offers some benefits for its users, but if its complexity outweighs these benefits, then the system is no longer worthwhile. Distributed consensus is a problem that is fundamentally complex, and a large chunk of its complexity budget must be spent just to arrive at a complete and working solution. I think many consensus algorithms and systems before Raft have exhausted their complexity budgets, and this might explain why few consensus-based systems are readily available. I hope Raft has changed this calculation and made these systems worth building.

12.1.2 On bridging theory and practice

We started this work because we wanted to build a system using consensus and found that it was surprisingly hard to do. This resonated with many others that had tried consensus and had given up on it in the past, but its value was lost on many academics. By making things simple and obvious, Raft appears almost uninteresting to academics. The academic community has not considered understandability per se to be an important contribution; they want novelty in some other dimension.
Academia should be more open to work that bridges the gap between theory and practice. This type of work may not bring any new functionality in theory, but it does give a larger number of students and practitioners a new capability, or at least substantially reduces their burden. The question of “Would I teach, use, and recommend this work?” is too often ignored, when, ultimately, it matters to our field.

The task of bridging the gap often needs to come from academic research. In industry, deadlines to ship products usually lead practitioners to ad hoc solutions that are just good enough to meet their needs. They can point out challenges (as the Chubby authors did with Paxos [15]), but they cannot usually invest the time needed to find the best solutions. With Raft, we weren’t content with good enough, and we think that is what makes our work valuable. We explored all the design choices we could think of; this took careful study at a depth that is difficult to accommodate in industry, but it produced a valuable result that many others can benefit from.

12.1.3 On finding research problems

When I started graduate school, I did not know how to find interesting research problems to work on. This seems silly to me now, as there are too many problems out there. There are various approaches to finding them, but I have found this one to be effective:

- First, start building something. I do not think it matters much what this something is, as long as you are motivated to build it. For example, you might choose to build an application you would like to have or rewrite an existing project in a new programming language you would like to learn.

- Second, pick a metric and optimize your system for it. For Raft, we set out to design the most understandable algorithm. Other projects optimize for performance, security, correctness, usability, or a number of other metrics.

The key to this approach is to ask, at every step of the way, what is the absolute best possible way to maximize your metric? This inevitably leads to either discovering something new to learn, or quite often, finding that no existing solution is quite good enough—a potential research project.

The problem then shifts from not having any problems to work on to having too many, and the challenge becomes deciding which one(s) to choose. This can pose a difficult judgment call; I suggest looking for projects that are conceptually interesting, are exciting to work on, and have the potential for significant impact.
12.2 Final comments

This dissertation aims to bridge the gap between theory and practice in distributed consensus. Much of the prior academic work on distributed consensus has been theoretical in nature and difficult to apply to building practical systems. Meanwhile, many of the real-world systems based on consensus have been ad hoc in nature, where practitioners have stopped at solutions that were good enough for their needs, and their implications and alternatives were not fully explored. In Raft, we have thoroughly explored the design space for a complete consensus algorithm with a focus on understandability, and we have also built a complete consensus-based system in order to ensure that our ideas are practical. We hope this will serve as a good foundation both for teaching consensus and for building future systems.
Appendix A

User study materials

This appendix includes various materials used in the Raft user study (Chapter 7):

- Section A.1 contains the Raft quiz questions, answers, and grading rubric.
- Section A.2 contains the Paxos quiz questions, answers, and grading rubric.
- Section A.3 contains the survey and the open-ended comments and feedback received from the participants.
- Section A.4 contains the summaries of the Raft and Paxos algorithms made available to participants during the study.

A.1 Raft quiz

Grading note: Where points are taken away for incorrect information, every section of every question still has a minimum of 0 points.

1. (4 points) Each figure below shows a possible log configuration for a Raft server (the contents of log entries are not shown; just their indexes and terms). Considering each log in isolation, could that log configuration occur in a proper implementation of Raft? If the answer is “no,” explain why not.

(a)

<table>
<thead>
<tr>
<th>log index</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>term</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>
**Answer:** No: terms increase monotonically in a log.
Specifically, the leader that created entry (4,2) could only have received (3,3) from a leader with current term \( \geq 3 \), so its current term would also be \( \geq 3 \). Then it could not create (4,2).

(b)  

![Log Index and Term Diagram]

**Answer:** Yes

(c)  

![Log Index and Term Diagram]

**Answer:** Yes

(d)  

![Log Index and Term Diagram]

**Answer:** No: logs cannot have holes.
Specifically, leaders only append to their logs, and the consistency check in Append-Entries never matches a hole.

**Grading:** 4 points total
One point per part.
If the answer is yes, saying “yes” earns 1 point. Saying “no” earns no points. Any supporting explanations are ignored.
If the answer is no, saying “no” earns half of the point, and a correct explanation earns the other half. Not much supporting explanation is required. Saying “yes” earns no points, and any accompanying explanation is ignored.

2. (6 points) The figure below shows the state of the logs in a cluster of 5 servers (the contents of the entries are not shown). Which log entries may safely be applied to state machines? Explain your answer.
Answer: Entries (1,1) and (2,1) may be safely applied:

If an entry is not stored on a quorum, it cannot be applied safely. This is because this minority can fail, and the other servers (which form a majority) can proceed with no knowledge of the entry.

Thus, we need only consider entries (1,1), (2,1), (3,2), (4,2), (5,2).

We need to figure out which ones could be elected leader, and see if they could cause these entries to be removed.

Server 2 can be elected leader because its log is at least as complete as S3, S4, and S5. It could then cause servers to remove entries (3,2), (4,2), and (5,2), so those entries are not safe to apply.

So now we’re left with entries (1,1), (2,1) as possibly safe to apply.

Servers 3 and 4 can’t be elected leader because their logs are not complete enough. Server 5 can be elected leader, but it contains (1,1) and (2,1).

Therefore, only entries (1,1) and (2,1) are safe to apply.

Grading: 6 points total

3 points for saying “entries (1,1) and (2,1)” or “entries 1 and 2” (since there is no ambiguity). No partial credit is awarded for these 3 points, but responses with an incorrect answer may still be awarded partial credit for the explanation.

3 points for the explanation:

+ 1 point for saying the entry must be stored on a quorum

+ 2 points for saying that server 2 may be elected leader, which threatens entries past index 2.

An answer that says “1 and 2 because entries from term 2 can’t be committed until one of the entries from the leader’s term reaches a majority of servers” receives 4.5 points (we got
3 answers like this; it's correct but not clear whether the participants understood why.
The incorrect answer of “entries 1–5 because they are stored on a majority” gets 1 point.
The incorrect answer of “entries 1–6 because they are stored on a majority” gets 0 points (entry 6 is not).

3. (10 points) Consider the figure below, which displays the logs in a cluster of 6 servers just after a new leader has just been elected for term 7 (the contents of log entries are not shown; just their indexes and terms). For each of the followers in the figure, could the given log configuration occur in a properly functioning Raft system? If yes, describe how this could happen; if no, explain why it could not happen.

**Answer:**

a) No. Entry (5,3) uniquely identifies a log prefix (by the AppendEntries consistency check), but this follower has entry (5,3) and a different log prefix before it than the leader.

b) No. Entry (6,5) uniquely identifies a log prefix (by the AppendEntries consistency check), but this follower has entry (6,5) and a different log prefix before it than the leader.

c) Yes. Since we can’t say much about the other servers in the cluster, this server could have been leader in term 6 with a starting log of (1,1), (2,1) and could have written a bunch of entries to its log and not communicated with our current leader of term 7. This assumes that entries (3,3), (4,3), (5,3), and (6,5) were not committed in term 5, which is possible.

d) No. Terms increase monotonically in a log. Specifically, the leader that created entry (5,2) could only have received (4,3) from a leader with current term ≥ 3, so its current term would also be ≥ 3. Then it could not create (5,2).
e) Yes. For example, (e) is the leader for term 1 and commits entries (1,1) and (2,1), then becomes partitioned from the other servers but continues processing client requests.

**Grading:** 10 points total
Two points per part:
+ 1 for the boolean,
+ 1 for a correct explanation.

If the boolean is incorrect, no points are awarded for the explanation.
If the boolean is correct, not much supporting explanation is required.

4. (5 points) Suppose that a hardware or software error corrupts the nextIndex value stored by the leader for a particular follower. Could this compromise the safety of the system? Explain your answer briefly.

**Answer:** No.
If the nextIndex value is too small, the leader will send extra AppendEntries requests. Each will have no effect on the follower’s log (they will pass the consistency check but not conflict with any entries in the follower’s log or provide any entries to the follower that the follower didn’t already have), and the successful response will indicate to the leader that it should increase its nextIndex.

If the nextIndex value is too large, the leader will also send extra AppendEntries requests. The consistency check will fail on these, causing the follower to reject the request and the leader to decrement nextIndex and retry.

Either way, this is safe behavior, as no critical state is modified in either case.

**Grading:** 5 points total
+ 1 point for saying “no”.
+ 2 points for explaining what happens if nextIndex is too small.
+ 2 points for explaining what happens if nextIndex is too large.

Answers that say a follower would truncate its log when nextIndex is too small receive -1 points, as that could result in a safety violation.

If the boolean is incorrect, partial credit may still be awarded for correct explanations.

5. (5 points) Suppose that you implemented Raft and deployed it with all servers in the same datacenter. Now suppose that you were going to deploy the system with each server in a different datacenter, spread over the world. What changes would you need to make, if any, in the wide-area version of Raft compared to the single-datacenter version, and why?
Answer: We’d need to set the election timeouts higher: the expected broadcast time is higher, and the election timeout should be much higher than the broadcast time so that candidates have a chance to complete an election before timing out again. The rest of the algorithm does not require any changes, since it does not depend on timing.

Grading: 5 points total
For full credit, an answer needs to include increasing the election timeout and as justification mention increased latency or some sort of livelock.
Answers that talk about “increasing timeouts” without specifically mentioning elections receive up to 3.5 points (this affects 4 answers).
Unnecessary or optional changes (performance improvements) are ignored if correctly identified as such.
Negative points are awarded for other changes identified as required.

6. (10 points) Each follower stores 3 pieces of information on its disk: its current term, its most recent vote, and all of the log entries it has accepted.

(a) Suppose that the follower crashes, and when it restarts, its most recent vote has been lost. Is it safe for the follower to rejoin the cluster (assuming no modifications to the algorithm)? Explain your answer.
Answer: No. This would allow a server to vote twice in the same term, which would then allow multiple leaders per term, which breaks just about everything.
For example, with 3 servers:
S1 acquires S1 and S2’s votes and becomes leader of term 2.
S2 restarts and forgets it voted in term 2.
S3 acquires S2 and S3’s votes and becomes the second leader of term 2.
Now S1 and S3 can commit distinct entries in term 2 with the same index and terms but different values.

(b) Now suppose that the follower’s log is truncated during a crash, losing some of the entries at the end. Is it safe for the follower to rejoin the cluster (assuming no modifications to the algorithm)? Explain your answer.
Answer: No. This would allow a committed entry to not be stored on a quorum, which would then allow some other entry to be committed for the same index.
For example, with 3 servers.
S1 becomes leader in term 2 and appends index=1, term=2, value=X on itself and S2. S1 sets its committedIndex to 1 and returns to the client that X is committed.
S2 restarts and loses the entry from its log.
S3 (with an empty log) becomes leader in term 3, since its empty log is at least as complete as S2’s. S3 appends index=1, term=3, value=Y on itself and S2. S3 sets its committedIndex to 1 and returns to the client that Y is committed.

Grading: 10 points total
5 points per part:
+ 1 point for the boolean,
+ 4 points for a correct explanation (the detailed scenarios above are not required)
For full credit on part (a), answers needed to include that this would allow multiple leaders to be elected for the same term, not just that a follower could vote twice.
If the boolean is incorrect, no points are awarded for the explanation.

7. (10 points) As described in the video, it’s possible for a leader to continue operating even after other servers have decided that it crashed and elected a new leader. The new leader will have contacted a majority of the cluster and updated their terms, so the old leader will step down as soon as it communicates with any of these servers. However, in the meantime it can continue to act as leader and issue requests to followers that have not yet been contacted by the new leader; furthermore, clients may continue to send requests to the old leader. We know that the old leader cannot commit any new log entries it receives after the election has completed, since it would need to contact at least one of the servers in the electing majority to do this. But, is it possible for an old leader to execute a successful AppendEntries RPC that completes the commitment of an old log entry that was received before the election started? If so, explain how this could happen, and discuss whether or not this will cause problems for the Raft protocol. If this cannot happen, explain why.

Answer: Yes. This can only happen if the new leader also contains the entry being committed, so it will not cause problems.
Here’s an example of this happening with 5 servers:
S1 with an empty log becomes leader for term 2 with votes S1, S2, and S3. S1 completes appending index=1, term=2, value=X to itself and S2. S2 with index=1, term=2, value=X in its log becomes leader for term 3 with votes S2, S4, S5.
SI completes appending index=1, term=2, value=X to S3. At this point, SI has completed commitment of index=1, term=2, value=X, even though it is no longer the current leader.

This behavior is safe because any new leader must also contain the entry, and so it will persist forever:

The entry must be stored on some server S that votes for the new leader L, and it must be stored on S before S votes for that new leader. The log completeness check says that S may only vote for L if:

\[ L.lastLogTerm > S.lastLogTerm \text{ or } \]
\[ (L.lastLogTerm == S.lastLogTerm \text{ and } L.lastLogIndex >= S.lastLogIndex) \]

If L is the first leader after S, we must be in the second case, and then L must contain every entry that S has, including the one we’re worried about.

If L’ is the second leader after S, L’ could only have a larger last term than S if it received entries from L. But L must have replicated the entry we’re worried about to L’ prior to replicating any of its own entries to L’, so this is also safe.

And this argument holds inductively for all future leaders.

**Grading:** 10 points total

- 4 points for showing this is possible:
  + 1 point for saying “Yes, it is possible”
  + For the remaining 3 points, answers must include that the deposed leader completed an AppendEntries request to one of the voters of the new leader before that server voted.
- 6 points for arguing that it is not a problem:
  + 1 point for saying “It’s not a problem.”
  + For the remaining 5 points, answers must include that because some voter must have the entry, the log completeness check guarantees that the new leader must also have the entry.

No points awarded for saying this cannot happen.

Credit for the scenario may be awarded even if the answer argues that this is a problem for Raft.

8. (10 points) During configuration changes, if the current leader is not in \( C_{\text{new}} \), it steps down once the log entry for \( C_{\text{new}} \) is committed. However, this means that there is a period of time when the leader is not part of the cluster it’s leading (the current configuration entry stored on
the leader is \( C_{\text{new}} \), which does not include the leader. Suppose the protocol were modified so that the leader steps down as soon as it stores \( C_{\text{new}} \) in its log, if \( C_{\text{new}} \) doesn’t include the leader. What’s the worst that could happen with this approach?

**Answer:** Depending on the interpretation of the algorithm, there are two possible correct answers.

Answer 1 assumes an implementation wherein once a server is not part of its current configuration, it does not become candidate anymore. The problem is that another server in \( C_{\text{old}} \) could then be elected as leader, append \( C_{\text{new}} \) to its log, and immediately step down. Worse yet, this could repeat for a majority of the servers in \( C_{\text{old}} \). It couldn’t repeat more than that because once a majority of \( C_{\text{old}} \) stores the \( C_{\text{new}} \) entry, no server from \( C_{\text{old}} \) without this entry could be elected due to the log completeness check (a majority of \( C_{\text{old}} \) required for \( C_{\text{old,new}} \) would no longer grant its vote to this server).

After this, a server in \( C_{\text{new}} \) would have to get elected, and the cluster would continue. So the worst case is really just running through up to about \( |C_{\text{old}}|/2 \) extra elections and election timeouts.

Answer 2 assumes a naïve implementation that allows a server that is not part of its current configuration to still become candidate. In this case, the worst thing that could happen is that the leader gets elected again as soon as it steps down (its log is still complete), then steps down again, then repeats infinitely.

**Grading:** 10 points total

For full credit, an answer needs to identify that a server not in \( C_{\text{new}} \) can be elected, that this can repeat, include a reasonable bound on this repetition, and mention that this causes an availability or liveness problem.

### A.2 Paxos quiz

**Grading note:** Where points are taken away for incorrect information, every section of every question still has a minimum of 0 points.

1. (4 points) Each figure below shows a possible log configuration for a Multi-Paxos server (the number in each log entry gives its acceptedProposal value). Considering each log in isolation, could that log configuration occur in a proper implementation of Multi-Paxos?
APPENDIX A. USER STUDY MATERIALS

1. For each of the following scenarios, answer Yes or No:

   a) \( \text{acceptedProposal} \text{(round, serverId)} \)

   Yes or No?

   **Answer:** Yes

   b) \( \text{acceptedProposal} \text{(round, serverId)} \)

   Yes or No?

   **Answer:** Yes

   c) \( \text{acceptedProposal} \text{(round, serverId)} \)

   Yes or No?

   **Answer:** Yes

   d) \( \text{acceptedProposal} \text{(round, serverId)} \)

   Yes or No?

   **Answer:** Yes

   **Grading:** 1 point per boolean (no partial credit)

2. (6 points) In Basic Paxos, suppose that a cluster contains 5 servers and 3 of them have accepted proposal 5.1 with value X. Once this has happened, is it possible that any server in the
cluster could accept a different value Y? Explain your answer.

**Answer:** Yes. If it’s S1, S2, and S3 that have accepted (5.1, X), other servers could still accept Y if it has a stale proposal number.

For example, S4 could prepare 3.4 and discover no values. Then S1 could prepare 5.1 on just S1, S2, S3. Then S1 could complete accepts on just S1, S2, S3. And S4 can still complete accepts on S4 and S5 with (3.4, Y).

**Grading:** 6 points total
2 points for saying “Yes”, and 4 points for the accompanying explanation. The explanation must indicate that Y’s proposal is concurrent with or numbered less than 5.1 (otherwise, −2 points).

The incorrect answer “No, because any new proposal must discover (5.1, X) in its prepare phase” receives 2 points.

Other incorrect answers with “No” receive no credit.

3. (10 points) Suppose that a server has just decided to act as leader in Multi-Paxos, and that no other servers are currently acting as leaders. Furthermore, assume that the server continues as leader for a period of time, arranging for many commands to be chosen for log entries, and that no other server attempts to act as leader during this period.

   a) What is the lower bound on the number of rounds of Prepare RPCs that the server must issue during this period? Explain your answer, and be as precise as possible.

   **Answer:** The lower bound is 1 round of Prepare RPCs, if a quorum of Prepare responses are returned right away that have noMoreAccepted=true.

   b) What is the upper bound on the number of rounds of Prepare RPCs that the server must issue during this period? Explain your answer, and be as precise as possible.

   **Answer:** The upper bound is one round of Prepare RPCs for each slot that is not chosen on the leader for which any acceptor has accepted any proposal. This can happen if every time the leader issues a prepare for one of its unchosen slots, it discovers an acceptor that has already accepted some value; then it needs to adopt this value for this slot and continue trying with the next slot.

**Grading:** 10 points total
5 points per part
For part a:
+ 2 points for saying “I”
4. (5 points) When an acceptor is marking entries accepted using the firstUnchosenIndex provided by the proposer, it must first check the proposal number in the entries that it marks. Suppose it skipped this check: describe a scenario where the system would misbehave.

**Errata:** The question should have read “marking entries chosen” instead of “marking entries accepted”. The quizzes used in our study contained the error, which we did not notice until grading the responses.

**Answer:** The misbehavior that can arise is a server marking a value as chosen when a different value has been chosen. This requires a minimum of 2 competing proposals, 3 servers, and 2 log entries to show:

- **S1** completes a round of prepare for n=1.1, index=1 with S1, S2.
- **S1** completes only one accept for n=1.1, v=X, index=1 with S1 (itself).
- **S2** completes a round of prepare for n=2.2, index=1 with S2, S3 and gets back noMoreAccepted=true from both.
- **S2** completes a round of accept for n=2.2, v=Y, index=1 with S2, S3.
- **S2** marks index 1 as chosen.
- **S2** completes a round of accept for n=2.2, v=Z, index=2, firstUnchosenIndex=2 with S1, S2, and S3.

Here, **S1** would have misbehaved by setting n=1.1, v=X as chosen and applying X to its state machine. This is incorrect, since in fact Y was chosen.

**Grading:** 5 points total

Unfortunately, most of the answers were not as specific as we would have liked for the scenario.
Full credit required identifying that the previously accepted value was different from the chosen value on the proposer, and not just that the proposal number was different. This helps separate people that regurgitated the material from people that had some understanding of why the algorithm is the way it is. Answers missing this component received up to 4 points (typically 2–3), depending on how well they showed understanding.

Since we messed up the wording in the question, no points were taken off on this question for confusing the words “accepted” and “chosen” in the answer (answers were read with these words exchanged in any way possible to give the answer the maximum number of points).

5. (5 points) Suppose that the two parts of a proposal number (round number and unique server id) were exchanged, so that the server id is in the high-order bits.

   a) Would this compromise the safety of Paxos? Explain your answer briefly.

      Answer: No, since safety only requires proposals to be uniquely numbered (for a given index in Multi-Paxos). Because server IDs are unique to each server and round numbers still monotonically increase, this uniqueness is preserved.

   b) Would this compromise the liveness of Paxos? Explain your answer briefly.

      Answer: Yes, for example, the server with the largest ID could issue a Prepare RPC to every server in the cluster and then permanently fail. No other proposer would then be able to make any progress, since the remaining servers’ minProposal values would be too high for the remaining proposers.

**Grading:** 5 points total

+ 2 points for safety
+ 3 points for liveness

For safety, saying “no” is worth 1 point, and a correct explanation is worth 1 point. Not much supporting explanation is required. Saying “yes” earns no points, and any accompanying explanation is ignored.

For liveness, saying “yes” is worth 1 point, and a correct explanation is worth 2 points. Saying “no” earns no points, and any accompanying explanation is ignored.

6. (10 points) Suppose that a proposer executes the Basic Paxos protocol with an initial value of v1, but that it crashes at some (unknown) point during or after the execution of the protocol. Suppose that the proposer restarts and reexecutes the protocol from the beginning with the same proposal number used previously, but with a different initial value of v2. Is this safe? Explain your answer.
Answer: No. Different proposals must have distinct proposal numbers. Here's an example of something bad that can happen using 3 servers:

S1 completes Prepare(n=1.1) with S1, S2.
S1 completes Accept(n=1.1, v=v1) with S1.
S1 restarts.
S1 completes Prepare(n=1.1) with S2, S3 (and discovers no accepted proposals).
S1 completes Accept(n=1.1, v=v2) with S2, S3.
S1 responds to the client that v2 has been chosen.
S2 completes Prepare(n=2.2) with S1, S2 and gets back:
from S1: acceptedProposal=1.1, acceptedValue=v1,
from S2: acceptedProposal=1.1, acceptedValue=v2,
S2 chooses to use v1 arbitrarily.
S2 completes Accept(n=2.2, v=v1) with S1, S2, S3.
S2 responds to some client that v1 was chosen.

A different problem that can occur involves a request from before the crash being delivered after the crash:

S1 completes Prepare(n=1.1) with S1, S2.
S1 completes Accept(n=1.1, v=v1) with S1.
S1 sends Accept(n=1.1, v=v1) to S2 and S3, but they don’t receive it yet.
S1 restarts.
S1 completes Prepare(n=1.1) with S2, S3 (and discovers no accepted proposals).
S1 completes Accept(n=1.1, v=v2) with S2, S3.
S1 responds to the client that v2 has been chosen.
Now S2 and S3 receive the Accept(n=1.1, v=v1) request and overwrite their acceptedValue to be v1.
The state of the cluster is now that v1 is chosen, even though a client has been told that v2 was chosen.

Grading: 10 points total
2 points for saying “no”, and 8 points for a correct explanation
For full credit, answers needed to explain that v2’s prepare phase did not discover v1 and include some violation of safety.
Saying “yes” earns no points, and any accompanying explanation is ignored.
7. (10 points) In a successful Accept RPC the acceptor sets its minProposal to n (the proposal number in the Accept RPC). Describe a scenario where this actually changes the value of minProposal (i.e., minProposal isn’t already equal to n). Describe a scenario where the system would behave incorrectly without this code.

Answer: Working backwards, we need a server to receive an Accept that did not receive a Prepare, since otherwise its minProposal would be up to date. And for this to matter, a subsequent Accept needs to incorrectly not be rejected.

Using Basic Paxos and 5 servers.
S1 completes Prepare(n=1.1) with S1, S2, S3 (and discovers no accepted proposals).
S5 completes Prepare(n=2.5) with S3, S4, S5 (and discovers no accepted proposals).
S5 completes Accept(n=2.5, v=X) with S2, S3, S5. This is where S2’s minProposal would be to 2.5 upon processing the Accept request.
S5 returns to the client that X is chosen.
S1 completes Accept(n=1.1, v=Y) with S2. This would normally be rejected, but would be accepted if S2’s minProposal was not updated during Accept.
S3 completes Prepare(n=3.3) with S1, S2, S4 (and discovers n=1.1, v=Y).
S3 completes Accept(n=3.3, v=Y) with S1, S2, S3, S4, S5.
S3 returns to a client that Y is chosen.

Grading: 10 points total
+ 4 points for the first three steps showing how minProposal can be set during Accept.
+ 6 points for showing how the system misbehaves. For full credit, this must include a safety violation.

8. (10 points) Consider a configuration change in Multi-Paxos, where the old configuration consists of servers 1, 2, and 3, and the new configuration consists of servers 3, 4, and 5. Suppose that the new configuration has been chosen for entry N in the log, and entries N through N+α (inclusive) have also been chosen. Suppose that at this point the old servers 1 and 2 are shut down because they are not part of the new configuration. Describe a problem that this could cause in the system.

Answer: This could cause a liveness problem for the new cluster because firstUnchosenIndex on those servers may be less than N+α.

For example in the worst case, server 3 might have failed permanently, and servers 1 and 2 would have made no attempt to transfer any values to servers 4 and 5 (using just the algorithm
presented in the lecture). Then, try as they might, servers 4 and 5 will never be able to learn the chosen values for slots 1 through $N+\alpha-1$ (inclusive), since they can’t communicate with servers 1, 2, or 3. Server 4 and 5’s state machines would never be able to advance beyond their initial state.

**Grading:** 10 points total
A complete answer must say that the new servers are missing chosen entries and dismiss server 3 as the solution.

Answers received up to 7 points if they implied server 3 must have all information (it can fail).
Answers received up to 8 points if they implied server 3 having all information is sufficient (it can fail).

No points are awarded for incorrectly saying there is no problem.
No points are awarded for incorrectly saying that some slots in the range 1 through $N-1$ (inclusive) may not have been chosen. That’s because $N$ through $N+\alpha$ (inclusive) chosen implies 1 through $N+\alpha$ (inclusive) are chosen by the definition of $\alpha$.

### A.3 Survey

1. Please rate any prior exposure you’ve had to Paxos.
   - I had never seen it before
   - I had seen it before but didn’t remember it
   - I had seen it before but remembered only a little bit
   - I had seen it before and remembered quite a bit
   - I had seen it before and consider myself an expert

**Responses:** Responses are presented in Figure 7.9.

2. Please rate any prior exposure you’ve had to Raft.
   - I had never seen it before
   - I had seen it before but didn’t remember it
   - I had seen it before but remembered only a little bit
   - I had seen it before and remembered quite a bit
   - I had seen it before and consider myself an expert

**Responses:** No participants reported having seen Raft before.
3. Do you think the video lectures were roughly equal in quality, given the nature of the material being presented?

- Paxos lecture was much better
- Paxos lecture was somewhat better
- They were roughly equal
- Raft lecture was somewhat better
- Raft lecture was much better

**Responses:** Responses are presented in Figure 7.10.

4. Do you think the quizzes were roughly equal in terms of testing your understanding of the material?

- Paxos questions were unfairly hard
- Paxos questions were somewhat harder
- They were roughly equal
- Raft questions were somewhat harder
- Raft questions were unfairly hard

**Responses:** Responses are presented in Figure 7.10.

5. Suppose you were working at a company and it is your job to implement a replicated state machine. Which algorithm would be easier to implement in a functioning, correct, and efficient system?

- Paxos would be much easier
- Paxos would be somewhat easier
- They would be roughly equal
- Raft would be somewhat easier
- Raft would be much easier

**Responses:** Responses are presented in Figure 7.11.

6. Suppose you had to explain either Raft or Paxos to a CS graduate student who hadn’t seen either one previously. Which would be easier to explain?

- Paxos would be much easier
- Paxos would be somewhat easier
- They would be roughly equal
• Raft would be somewhat easier
• Raft would be much easier

Responses: Responses are presented in Figure 7.11.

7. Do you have any additional comments?

Responses: The participants’ responses are reproduced below in random order, exactly as submitted (errors included):

• I was forced to go back and re-watch parts of the Paxos lecture in order to answer the quiz questions. I could answer most of the Raft questions from memory.
• I started a bit late on watching the videos. Without as much time to fully absorb the material before taking the quizzes, I left a couple of parts incomplete.
  I liked the one-slide summary near the beginning of the Raft lecture.
• Both are super complex!
• Good job on the lecture videos.
• Raft might be simpler, but the lecture on it was much harder to understand. In particular, the requirements for each step (leader election, or considering when an entry is committed) were incrementally built and scattered through the lecture, which made it really hard for me to fit the whole thing together mentally. In other words, I got each chunk of the protocol and why certain checks had to be there, but I had no ability to put the whole thing together and get the big picture, which was what I had to do on the quiz, because the quiz was asking me to synthesize and predict Raft’s behavior, and understand how the checks (like on committing and leader elections) interacted with each other.
• I have the vague feeling raft and paxos are too much alike. maybe even the same, but I can’t tell because I don’t fully understand paxos.
• It appears that Raft is equivalent to Multi-Paxos, yet Multi has not been proven or implemented? I’m a little confused how Paxos is used in practice.
• Cool idea, I think it’s what Paxos should have been.
• Raft felt easier that MultiPaxos. Reasoning about possible log states for Multi Paxos felt tougher for me than Raft.
  There were couple of minor things not quite clear to me. One is about an optimization, when a leader tries to catch up the log on a follower (who is way behind) it goes one
entry at a time till it finds a match. May be this process could be speeded up if the follower responds with its last entry (or exchange multiple entries separated by some distance so that the number of round trips can be reduced).

May be this sort of opt was left out for the sake of simplicity.

Second point was about Cnew+old (I assume that this is the union of machines in Cold and Cnew). Also when a follower assumes the new configuration, and if he is not in it, does he take himself off? We discussed the leader case in the lecture.

- I obviously noticed that Raft and Paxos are very similar - to the point that I feel like Raft is actually paxos presented differently. But I definitely found Raft to be easier to grasp conceptually, explain, and implement. Although I do think that if each piece in Paxos is presented more strategically like Raft, the differences would become much less apparent.

- The quizzes were too long. Could not complete in the time provided. Also, with just an hr of lecture its difficult to answer the questions in the quiz, given that I have never seen anything even close to this before. A set of examples apart from the video lecture would have been helpful.

- Raft is much easier conceptually but I’m curious about how commonly and effectively it is implemented. Paxos is more popular, so I would expect it to have more reliable implementation. However, I hope Raft gains more popularity and becomes the mainstream distributed systems consensus protocol.

- Paxos is easier to understand because it does not have any many details as Raft. But the video of Paxos does not help to the questions as much as the video of Raft do.

- Ousterhout is a boss. Thanks for the lectures!

- Is it just me or is Raft far easier to understand, especially due to its leader-follower nature? The only distributed decision there is leader election and that is easy, as compared to Paxos where everything is a distributed decision and where logs can get messy and complicated. Both are fine algorithms, though I would prefer Raft if I ever had to use either (depending on real-world performance, at which Raft would presumably be better).

- Just the last portion on configuration wasn’t too clear. Everything else was conceptually easier.
• I took the raft quiz first. After seeing how elegantly raft solved the consensus problem, paxos approach seems to be filled with a lot of unnecessary complexity.

• Videos were a bit dry

A.4 Supporting materials

Figure A.1 shows the Raft algorithm summary made available to user study participants as part of the Raft lecture slides. Figures A.2 through A.5 show the Paxos summary made available to participants as a separate document on the study web site.
## Raft Protocol Summary

### Followers
- Respond to RPCs from candidates and leaders.
- Convert to candidate if election timeout elapses without either:
  - Receiving valid AppendEntries RPC, or
  - Granting vote to candidate

### Candidates
- Increment currentTerm, vote for self
- Reset election timeout
- Send RequestVote RPCs to all other servers, wait for either:
  - Votes received from majority of servers: become leader
  - AppendEntries RPC received from new leader: step down
- Election timeout elapses without election resolution: increment term, start new election
- Discover higher term: step down

### Leaders
- Initialize nextIndex for each to last log index + 1
- Send initial empty AppendEntries RPCs (heartbeat) to each follower; repeat during idle periods to prevent election timeouts
- Accept commands from clients, append new entries to local log
- Whenever last log index ≥ nextIndex for a follower, send AppendEntries RPC with log entries starting at nextIndex, update nextIndex if successful
- If AppendEntries fails because of log inconsistency, decrement nextIndex and retry
- Mark log entries committed if stored on a majority of servers and at least one entry from current term is stored on a majority of servers
- Step down if currentTerm changes

### Persistent State
Each server persists the following to stable storage synchronously before responding to RPCs:
- `currentTerm`: latest term server has seen (initialized to 0 on first boot)
- `votedFor`: candidateId that received vote in current term (or null if none)
- `log[]`: log entries

### Log Entry
- `term`: term when entry was received by leader
- `index`: position of entry in the log
- `command`: command for state machine

### RequestVote RPC
- Invoked by candidates to gather votes.
- **Arguments:**
  - `candidateId`: candidate requesting vote
  - `term`: candidate's term
  - `lastLogIndex`: index of candidate's last log entry
  - `lastLogTerm`: term of candidate's last log entry
- **Results:**
  - `term`: currentTerm, for candidate to update itself
  - `voteGranted`: true means candidate received vote
- **Implementation:**
  1. If `term > currentTerm`, currentTerm ← term (step down if leader or candidate)
  2. If `term == currentTerm`, votedFor is null or candidateId, and candidate's log is at least as complete as local log, grant vote and reset election timeout

### AppendEntries RPC
- Invoked by leader to replicate log entries and discover inconsistencies; also used as heartbeat.
- **Arguments:**
  - `term`: leader's term
  - `leaderId`: so follower can redirect clients
  - `prevLogIndex`: index of log entry immediately preceding new ones
  - `prevLogTerm`: term of prevLogIndex entry
  - `entries[]`: log entries to store (empty for heartbeat)
  - `commitIndex`: last entry known to be committed
- **Results:**
  - `term`: currentTerm, for leader to update itself
  - `success`: true if follower contained entry matching prevLogIndex and prevLogTerm
- **Implementation:**
  1. Return if term < currentTerm
  2. If term > currentTerm, currentTerm ← term
  3. If candidate or leader, step down
  4. Reset election timeout
  5. Return failure if log doesn’t contain an entry at prevLogIndex whose term matches prevLogTerm
  6. If existing entries conflict with new entries, delete all existing entries starting with first conflicting entry
  7. Append any new entries not already in the log
  8. Advance state machine with newly committed entries

---

Figure A.1: Raft summary used in the user study. This is an earlier version of Figure 3.1.
Paxos summary

Diego Ongaro and John Ousterhout

March 6, 2013

This document provides a terse summary of the Basic Paxos (single-decree) consensus protocol as well as Multi-Paxos. It is intended as an accompaniment to a one-hour video lecture introducing Paxos, which was developed as part of a user study comparing Paxos with the Raft consensus algorithm. Multi-Paxos is not specified precisely in the literature; our goal here is to provide a fairly complete specification that stays close to Leslie Lamport’s original description of Paxos in “The Part-Time Parliament.” The version of Multi-Paxos described here has not been implemented or proven correct.

1 Basics

- proposal number \( (n) = (\text{round number}, \text{server ID}) \)
- \( T \): a fixed timeout value used in the leader election algorithm
- \( \alpha \): concurrency limit in Multi-Paxos

1.1 Leader election algorithm

- Every \( T \) milliseconds, send an empty heartbeat message to every other server.
- A server acts as leader if it has not received a heartbeat message in the last \( 2T \) milliseconds from a server with higher ID.

2 Basic Paxos (Single-decree)

2.1 Persistent state per server

- \( \text{minProposal} \): the number of the smallest proposal this server will accept, or 0 if it has never received a Prepare request
- \( \text{acceptedProposal} \): the number of the last proposal the server has accepted, or 0 if it never accepted any
- \( \text{acceptedValue} \): the value from the most recent proposal the server has accepted, or null if it has never accepted a proposal
- \( \text{maxRound} \): the largest round number the server has seen

2.2 Messages

2.2.1 Prepare (Phase 1)

Request fields:
- \( n \): a new proposal number

Upon receiving a Prepare request, if \( n \geq \text{minProposal} \), the acceptor sets \( \text{minProposal} \) to \( n \). The response constitutes a promise to reject Accept messages with proposal numbers less than \( n \) in the future.

Response fields:
- \( \text{acceptedProposal} \): the acceptor’s \( \text{acceptedProposal} \)
- \( \text{acceptedValue} \): the acceptor’s \( \text{acceptedValue} \)
2.2.2 Accept (Phase 2)

Request fields:
- \( n \): the same proposal number used in Prepare
- \( v \): a value, either the highest numbered one from Prepare responses, or if none, then one from a client request

Upon receiving an Accept request, if \( n \geq \text{minProposal} \), then:
- Set \( \text{acceptedProposal} = n \)
- Set \( \text{acceptedValue} = v \)
- Set \( \text{minProposal} = n \)

Response fields:
- \( n \): the acceptor’s \( \text{minProposal} \)

2.3 Proposer Algorithm: \( \text{write}(\text{inputValue}) \rightarrow \text{chosenValue} \)

1. Let \( n \) be a new proposal number (increment and persist \( \text{maxRound} \)).
2. Broadcast \( \text{Prepare}(n) \) requests to all acceptors.
3. Upon receiving \( \text{Prepare} \) responses \( (\text{reply.acceptedProposal}, \text{reply.acceptedValue}) \) from a majority of acceptors:
   - Let \( v \) be set as follows: if the maximum \( \text{reply.acceptedProposal} \) in the replies isn’t 0, use its corresponding \( \text{reply.acceptedValue} \). Otherwise, use \( \text{inputValue} \).
4. Broadcast \( \text{Accept}(n,v) \) requests.
5. Upon receiving an \( \text{Accept} \) response with \( (\text{reply.n}) \):
   - If \( \text{reply.n} > n \), set \( \text{maxRound} \) from \( n \), and start over at step 1.
6. Wait until receiving \( \text{Accept} \) responses for \( n \) from a majority of acceptors.
7. Return \( v \).

3 Multi-Paxos

3.1 Persistent state per acceptor

Each acceptor stores:
- \( \text{lastLogIndex} \): the largest entry for which this server has accepted a proposal
- \( \text{minProposal} \): the number of the smallest proposal this server will accept for any log entry, or 0 if it has never received a Prepare request. This applies globally to all entries.

Each acceptor also stores a log, where each log entry \( i \in [1, \text{lastLogIndex}] \) has the following fields:
- \( \text{acceptedProposal}[i] \): the number of the last proposal the server has accepted for this entry, or 0 if it never accepted any, or \( \infty \) if \( \text{acceptedValue}[i] \) is known to be chosen
- \( \text{acceptedValue}[i] \): the value in the last proposal the server accepted for this entry, or null if it never accepted any

Define \( \text{firstUnchosenIndex} \) as the smallest log index \( i > 0 \) for which \( \text{acceptedProposal}[i] < \infty \)

3.2 Persistent state per proposer

- \( \text{maxRound} \): the largest round number the proposer has seen

3.3 Soft (volatile) state per proposer

(I’m not doing a very strong separation here between the proposer and the acceptor. I allow proposers to both read and write into acceptor state sometimes.)
- \( \text{nextIndex} \): the index of the next entry to use for a client request
- \( \text{prepared} \): True means there is no need to issue \( \text{Prepare} \) requests (a majority of acceptors has responded to \( \text{Prepare} \) requests with \( \text{noMoreAccepted} \) true); initially false

Figure A.3: Paxos summary used in the user study, page 2 of 4.
3.4 Messages

3.4.1 Prepare (Phase 1)

Request fields:
- \( n \): a new proposal number
- \( \text{index} \): the log entry that the proposer is requesting information about

Upon receiving a Prepare request, if \( \text{request.}n \geq \text{minProposal} \), the acceptor sets \( \text{minProposal} \) to \( \text{request.}n \).

The response constitutes a promise to reject Accept requests (for any log entry) with proposals numbered less than \( \text{request.}n \).

Response fields:
- \( \text{acceptedProposal} \): the acceptor’s \( \text{acceptedProposal}[\text{index}] \)
- \( \text{acceptedValue} \): the acceptor’s \( \text{acceptedValue}[\text{index}] \)
- \( \text{noMoreAccepted} \): set to true if this acceptor has never accepted a value for a log entry with index greater than \( \text{index} \)

3.4.2 Accept (Phase 2)

Request fields:
- \( n \): the same proposal number used in the most recent Prepare
- \( \text{index} \): identifies a log entry
- \( v \): a value, either the highest numbered one from a Prepare response, or if none, then one from a client request
- \( \text{firstUnchosenIndex} \): the sender’s \( \text{firstUnchosenIndex} \)

Upon receiving an Accept request: if \( n \geq \text{minProposal} \), then:

- Set \( \text{acceptedProposal}[\text{index}] = n \)
- Set \( \text{acceptedValue}[\text{index}] = v \)
- Set \( \text{minProposal} = n \)

For every \( \text{index} < \text{request.} \text{firstUnchosenIndex} \), if \( \text{acceptedProposal}[\text{index}] = n \), set \( \text{acceptedProposal}[\text{index}] \) to \( \infty \).

Response fields:
- \( n \): the acceptor’s \( \text{minProposal} \)
- \( \text{firstUnchosenIndex} \): the acceptor’s \( \text{firstUnchosenIndex} \).

3.4.3 Success (Phase 3)

Request fields:
- \( \text{index} \): identifies a log entry
- \( v \): the chosen value for entry \( \text{index} \)

Upon receiving a Success request, set \( \text{acceptedValue}[\text{index}] \) to \( v \) and \( \text{acceptedProposal}[\text{index}] = \infty \).

Response fields:
- \( \text{firstUnchosenIndex} \): the acceptor’s \( \text{firstUnchosenIndex} \)

When the sender receives the response, if \( \text{reply.} \text{firstUnchosenIndex} < \text{firstUnchosenIndex} \) then the sender sends \( \text{Success}(\text{index} = \text{reply.} \text{firstUnchosenIndex}, \text{value} = \text{acceptedValue}[\text{reply.} \text{firstUnchosenIndex}]) \).

3.5 Proposer Algorithm: \( \text{write}(\text{inputValue}) \to \text{bool} \)

1. If not leader or not done with leader initialization, return false.
2. If \( \text{prepared} \) is true:
   (a) Let \( \text{index} = \text{nextIndex} \), increment \( \text{nextIndex} \).
   (b) Go to step 6.
3. Let \( \text{index} = \text{firstUnchosenIndex} \) and \( \text{nextIndex} = \text{index} + 1 \).
4. Let \( n \) be a new proposal number (increment and persist \( \text{maxRound} \))
5. Broadcast \( \text{Prepare}(n, \text{index}) \) requests to all acceptors.
6. Upon receiving \( \text{Prepare} \) responses (\( \text{reply.} \text{acceptedProposal}, \text{reply.} \text{acceptedValue}, \text{reply.} \text{noMoreAccepted} \)) from a majority of acceptors:

Figure A.4: Paxos summary used in the user study, page 3 of 4.
Let $v$ be set as follows: if the maximum $\text{reply.acceptedProposal}$ in the replies isn’t 0, use its corresponding $\text{reply.acceptedValue}$. Otherwise, use $\text{inputValue}$.

- If all acceptors in the majority responded with $\text{reply.noMoreAccepted}$, set $\text{prepared} = \text{true}$.

7. Broadcast $\text{Accept}(\text{index}, n, v)$ requests to all acceptors.

8. Upon receiving an $\text{Accept}$ response with $(\text{reply.n}, \text{reply.firstUnchosenIndex})$:
   - If $\text{reply.n} > n$, set $\text{maxRound}$ from $\text{reply.n}$. Set $\text{prepared} = \text{false}$. Go to step 1.
   - If $\text{reply.firstUnchosenIndex} \leq \text{lastLogIndex}$ and $\text{acceptedProposal}[\text{reply.firstUnchosenIndex}] = \infty$, then send $\text{Success}(\text{index} = \text{reply.firstUnchosenIndex}, \text{value} = \text{acceptedValue}[\text{reply.firstUnchosenIndex}])$.

9. Upon receiving Accept responses for $n$ from a majority of acceptors:
   - Set $\text{acceptedProposal}[\text{index}] = \infty$ and $\text{acceptedValue[\text{index}]} = v$.

10. If $v == \text{inputValue}$, return true.

11. Go to step 2.

4 Reconfiguration

- Configuration is a list of ids and addresses of servers, stored as special log entries
- Configuration for choosing entry $i$ determined by latest configuration in log at entry $i - \alpha$ or below.
- $\alpha$ limits concurrency: can’t choose entry $i + \alpha$ until entry $i$ is chosen

Figure A.5: Paxos summary used in the user study, page 4 of 4.
Appendix B

Safety proof and formal specification

This appendix includes a formal specification and a proof of safety for the basic Raft algorithm presented in Chapter 3. The specification and proof are introduced in Chapter 8.

The formal specification makes the information summarized in Figure 3.1 completely precise using the TLA+ specification language [50]. It serves as the subject of the proof and is a useful reference for implementing Raft.

The proof shows that the specification preserves the State Machine Safety property. The main idea of the proof is summarized in Section 3.6.3, but the detailed proof is much more precise. We found the proof useful in understanding Raft’s safety at a deeper level, and others may find value in this as well. However, the proof is fairly long and difficult for humans to verify and maintain; we believe it to be basically correct, but it might include errors or omissions. At this scale, only a machine-checked proof could definitively be error-free.

B.1 Conventions

The specification uses the syntax and semantics of the TLA+ language version 2 [50]. The proof uses the same syntax and semantics but with the following minor allowances for convenience:

- As in TLA+, \( \text{foo}' \) has a specific meaning: the value of variable \( \text{foo} \) in the next state of the system.
- Define \( \langle \text{index}, \text{term} \rangle \in \log \) iff
  \[
  \text{Len}(\log) \geq \text{index} \land \log[\text{index}].\text{term} = \text{term}.
  \]
- The symbol \( \| \) is used for concatenation of logs and entries.
Values in log entries are not included, since a value is attached to a particular \( (index, term) \), and those uniquely identify a log entry.

### B.2 Specification

This section provides a complete, formal description of the Raft algorithm. A copy of the TLA+ source file can is available at [87].

```tla
MODULE raft

This is the formal specification for the Raft consensus algorithm.

It was last modified on July 6, 2014.

EXTENDS Naturals, FiniteSets, Sequences, TLC

The set of server IDs

CONSTANTS Server

The set of requests that can go into the log

CONSTANTS Value

Server states.

CONSTANTS Follower, Candidate, Leader

A reserved value.

CONSTANTS Nil

Message types:

CONSTANTS RequestVoteRequest, RequestVoteResponse,

AppendEntriesRequest, AppendEntriesResponse

Global variables

A bag of records representing requests and responses sent from one server

to another. TLAPS doesn’t support the Bags module, so this is a function

mapping Message to Nat.

VARIABLE messages

A history variable used in the proof. This would not be present in an
APPENDIX B. SAFETY PROOF AND FORMAL SPECIFICATION

VARIABLE elections
A history variable used in the proof. This would not be present in an implementation.

VARIABLE allLogs
Keeps track of every log ever in the system (set of logs).

The following variables are all per server (functions with domain Server).

VARIABLE currentTerm
The server’s term number.

VARIABLE state
The server’s state (Follower, Candidate, or Leader).

VARIABLE votedFor
The candidate the server voted for in its current term, or Nil if it hasn’t voted for any.

serverVars \Delta= \langle currentTerm, state, votedFor \rangle

VARIABLE log
A Sequence of log entries. The index into this sequence is the index of the log entry. Unfortunately, the Sequence module defines Head(s) as the entry with index 1, so be careful not to use that!

VARIABLE commitIndex
The index of the latest entry in the log the state machine may apply.

logVars \Delta= \langle log, commitIndex \rangle

The following variables are used only on candidates:

VARIABLE votesResponded
The set of servers from which the candidate has received a RequestVote response in its currentTerm.

VARIABLE votesResponded
The set of servers from which the candidate has received a vote in its
currentTerm.

VARIABLE votesGranted

A history variable used in the proof. This would not be present in an
implementation.

Function from each server that voted for this candidate in its currentTerm
to that voter’s log.

VARIABLE voterLog

candidateVars = \langle votesResponded, votesGranted, voterLog \rangle

The following variables are used only on leaders:

VARIABLE nextIndex

The latest entry that each follower has acknowledged is the same as the
leader’s. This is used to calculate commitIndex on the leader.

VARIABLE matchIndex

leaderVars = \langle nextIndex, matchIndex, elections \rangle

End of per server variables.

vars = \langle messages, allLogs, serverVars, candidateVars, leaderVars, logVars \rangle

Helpers

The set of all quorums. This just calculates simple majorities, but the only
important property is that every quorum overlaps with every other.

Quorum = \{ i \in\ \text{SUBSET} (Server) : \text{Cardinality}(i) \ast 2 > \text{Cardinality}(Server) \}\n
The term of the last entry in a log, or 0 if the log is empty.

LastTerm(xlog) = \text{IF} \ \text{Len}(xlog) = 0 \ \text{THEN} \ 0 \ \text{ELSE} \ xlog[\text{Len}(xlog)].term

Helper for Send and Reply. Given a message m and bag of messages, return a
new bag of messages with one more m in it.

WithMessage(m, msgs) = \text{IF} \ m \in \text{DOMAIN} \ \text{msgs} \ \text{THEN}
Helper for Discard and Reply. Given a message \( m \) and bag of messages, return a new bag of messages with one less \( m \) in it.

\[
\text{WithoutMessage}(m, \text{msgs}) \equiv \begin{cases} 
\begin{align*}
\text{msgs \ except \ !}[m] = \text{msgs}[m] + 1 \\
\text{ELSE} \\
\text{msgs} @ @ (m : > 1)
\end{align*}
\end{cases}
\]

Add a message to the bag of messages.

\[
\text{Send}(m) \equiv \text{messages}' = \text{WithMessage}(m, \text{messages})
\]

Remove a message from the bag of messages. Used when a server is done processing a message.

\[
\text{Discard}(m) \equiv \text{messages}' = \text{WithoutMessage}(m, \text{messages})
\]

Combination of Send and Discard

\[
\text{Reply}(\text{response}, \text{request}) \equiv \text{messages}' = \text{WithoutMessage}(\text{request}, \text{WithMessage}(\text{response}, \text{messages}))
\]

Return the minimum value from a set, or undefined if the set is empty.

\[
\text{Min}(s) \equiv \text{CHOOSE } x \in s \colon \forall y \in s : x \leq y
\]

Return the maximum value from a set, or undefined if the set is empty.

\[
\text{Max}(s) \equiv \text{CHOOSE } x \in s \colon \forall y \in s : x \geq y
\]

Define initial values for all variables

\[
\text{InitHistoryVars} \equiv \land \text{elections} = \{\} \\
\land \text{allLogs} = \{\} \\
\land \text{voterLog} = [i \in \text{Server} \mapsto [j \in \{\} \mapsto ø]]
\]

\[
\text{InitServerVars} \equiv \land \text{currentTerm} = [i \in \text{Server} \mapsto 1] \\
\land \text{state} = [i \in \text{Server} \mapsto \text{Follower}] \\
\land \text{votedFor} = [i \in \text{Server} \mapsto \text{Nil}]
\]
APPENDIX B. SAFETY PROOF AND FORMAL SPECIFICATION

\[ \text{InitCandidateVars} \triangleq \ \land \ \text{votesResponded} \; = \; [i \in \text{Server} \mapsto \{\}]
\]
\[ \land \ \text{votesGranted} \; = \; [i \in \text{Server} \mapsto \{\}]
\]

The values nextIndex[i][i] and matchIndex[i][i] are never read, since the
leader does not send itself messages. It’s still easier to include these
in the functions.

\[ \text{InitLeaderVars} \triangleq \ \land \ \text{nextIndex} \; = \; [i \in \text{Server} \mapsto [j \in \text{Server} \mapsto 1]]
\]
\[ \land \ \text{matchIndex} \; = \; [i \in \text{Server} \mapsto [j \in \text{Server} \mapsto 0]]
\]

\[ \text{InitLogVars} \triangleq \ \land \ \text{log} \; = \; [i \in \text{Server} \mapsto \langle \rangle]
\]
\[ \land \ \text{commitIndex} \; = \; [i \in \text{Server} \mapsto 0]
\]

\[ \text{Init} \triangleq \ \land \ \text{messages} \; = \; [m \in \{\} \mapsto 0]
\]
\[ \land \ \text{InitHistoryVars}
\]
\[ \land \ \text{InitServerVars}
\]
\[ \land \ \text{InitCandidateVars}
\]
\[ \land \ \text{InitLeaderVars}
\]
\[ \land \ \text{InitLogVars}
\]

Define state transitions

Server \textit{i} restarts from stable storage.

It loses everything but its currentTerm, votedFor, and log.

\[ \text{Restart}(i) \triangleq
\]
\[ \land \ \text{state}' \; = \; [\text{state} \ \text{EXCEPT} \ ![i] = \text{Follower}]
\]
\[ \land \ \text{votesResponded}' \; = \; [\text{votesResponded} \ \text{EXCEPT} \ ![i] = \{\}]
\]
\[ \land \ \text{votesGranted}' \; = \; [\text{votesGranted} \ \text{EXCEPT} \ ![i] = \{\}]
\]
\[ \land \ \text{voterLog}' \; = \; [\text{voterLog} \ \text{EXCEPT} \ ![i] = [j \in \{\} \mapsto \langle \rangle]]
\]
\[ \land \ \text{nextIndex}' \; = \; [\text{nextIndex} \ \text{EXCEPT} \ ![i] = [j \in \text{Server} \mapsto 1]]
\]
\[ \land \ \text{matchIndex}' \; = \; [\text{matchIndex} \ \text{EXCEPT} \ ![i] = [j \in \text{Server} \mapsto 0]]
\]
\[ \land \ \text{commitIndex}' \; = \; [\text{commitIndex} \ \text{EXCEPT} \ ![i] = 0]
\]
\[ \land \ \text{UNCHANGED} \langle \text{messages}, \text{currentTerm}, \text{votedFor}, \text{log}, \text{elections} \rangle
\]

Server \textit{i} times out and starts a new election.

\[ \text{Timeout}(i) \triangleq \ \land \ \text{state}[i] \in \{\text{Follower}, \text{Candidate}\}
\]
\[ \land \ \text{state}' \; = \; [\text{state} \ \text{EXCEPT} \ ![i] = \text{Candidate}]
\]
\[ \land \ \text{currentTerm}' \; = \; [\text{currentTerm} \ \text{EXCEPT} \ ![i] = \text{currentTerm}[i] + 1]
\]
Most implementations would probably just set the local vote atomically, but messaging localhost for it is weaker.

\[ \text{∧ votedFor}' = [\text{votedFor EXCEPT } !i = \text{Nil}] \]
\[ \text{∧ votesResponded}' = [\text{votesResponded EXCEPT } !i = \{\}] \]
\[ \text{∧ votesGranted}' = [\text{votesGranted EXCEPT } !i = \{\}] \]
\[ \text{∧ voterLog}' = [\text{voterLog EXCEPT } !i = [j \in \{\} \mapsto \langle \rangle]] \]
\[ \text{∧ UNCHANGED } \langle \text{messages, leaderVars, logVars} \rangle \]

Candidate $i$ sends $j$ a `RequestVote` request.

\[ \text{∧ state}[i, j] = \text{Candidate} \]
\[ \text{∧ j} \notin \text{votesResponded}[i] \]
\[ \text{∧ Send}([\text{mtype} \mapsto \text{RequestVoteRequest}, \text{mterm} \mapsto \text{currentTerm}[i], \text{mlastLogTerm} \mapsto \text{LastTerm}(\text{log}[i]), \text{mlastLogIndex} \mapsto \text{Len}(\text{log}[i]), \text{msource} \mapsto i, \text{mdest} \mapsto j]) \]
\[ \text{∧ UNCHANGED } \langle \text{serverVars, candidateVars, leaderVars, logVars} \rangle \]

Leader $i$ sends $j$ an `AppendEntries` request containing up to 1 entry.

While implementations may want to send more than 1 at a time, this spec uses just 1 because it minimizes atomic regions without loss of generality.

\[ \text{∧ i} \neq j \]
\[ \text{∧ state}[i] = \text{Leader} \]
\[ \text{∧ LET prevLogIndex} \triangleq \text{nextIndex}[i][j] - 1 \]
\[ \text{prevLogTerm} \triangleq \text{IF prevLogIndex} > 0 \text{ THEN} \]
\[ \text{log}[i][\text{prevLogIndex}].\text{term} \]
\[ \text{ELSE} \]
\[ 0 \]
\[ \text{Send up to 1 entry, constrained by the end of the log.} \]
\[ \text{lastEntry} \triangleq \text{Min}([\text{Len}(\text{log}[i]), \text{nextIndex}[i][j] + 1]) \]
\[ \text{entries} \triangleq \text{SubSeq}(\text{log}[i], \text{nextIndex}[i][j], \text{lastEntry}) \]
\[\text{IN} \ Send([mtype \mapsto \text{AppendEntriesRequest,}\]
\[\quad mterm \mapsto \text{currentTerm[i]},\]
\[\quad mprevLogIndex \mapsto \text{prevLogIndex},\]
\[\quad mprevLogTerm \mapsto \text{prevLogTerm},\]
\[\quad mentries \mapsto \text{entries},\]
\[\quad mlog \text{ is used as a history variable for the proof.}\]
\[\quad \text{It would not exist in a real implementation.}\]
\[\quad mlog \mapsto \text{log[i]},\]
\[\quad mcommitIndex \mapsto \text{Min}(...),\]
\[\quad msource \mapsto i,\]
\[\quad mdest \mapsto j)]\]
\[\land \text{UNCHANGED} \langle \text{serverVars, candidateVars, leaderVars, logVars} \rangle\]

\text{Candidate} i \text{ transitions to leader.}
\[\text{BecomeLeader}(i) \triangleq\]
\[\land \text{state}[i] = \text{Candidate}\]
\[\land \text{votesGranted}[i] \in \text{Quorum}\]
\[\land \text{state}' = [\text{state EXCEPT } ![i] = \text{Leader}]\]
\[\land \text{nextIndex}' = [\text{nextIndex EXCEPT } ![i] =\]
\[\quad [j \in \text{Server} \mapsto \text{Len}(\text{log}[i]) + 1]]\]
\[\land \text{matchIndex}' = [\text{matchIndex EXCEPT } ![i] =\]
\[\quad [j \in \text{Server} \mapsto 0]]\]
\[\land \text{elections}' = \text{elections} \cup\]
\[\{[\text{eterm} \mapsto \text{currentTerm}[i],\]
\[\quad \text{eleader} \mapsto i,\]
\[\quad \text{elog} \mapsto \text{log}[i],\]
\[\quad \text{evotes} \mapsto \text{votesGranted}[i],\]
\[\quad \text{evoterLog} \mapsto \text{voterLog}[i]]\}
\[\land \text{UNCHANGED} \langle \text{messages, currentTerm, votedFor, candidateVars, logVars} \rangle\]

\text{Leader} i \text{ receives a client request to add } v \text{ to the log.}
\[\text{ClientRequest}(i, v) \triangleq\]
\[\land \text{state}[i] = \text{Leader}\]
\[\land \text{LET entry} \triangleq [\text{term} \mapsto \text{currentTerm}[i],\]
\[\text{value} \mapsto v\]
\[\text{newLog} \triangleq \text{Append}(\log[i], \text{entry})\]
\[\text{IN } \log' = [\log \text{EXCEPT } ![i] = \text{newLog}]\]
\[\land \text{UNCHANGED} \langle \text{messages}, \text{serverVars}, \text{candidateVars}, \text{leaderVars}, \text{commitIndex} \rangle\]

Leader i advances its \text{commitIndex}.

This is done as a separate step from handling \text{AppendEntries} responses, in part to minimize atomic regions, and in part so that leaders of single-server clusters are able to mark entries committed.

\[\text{AdvanceCommitIndex}(i) \triangleq\]
\[\land \text{state}[i] = \text{Leader}\]
\[\land \text{LET } \text{The set of servers that agree up through index.}\]
\[\text{Agree}(\text{index}) \triangleq \{i\} \cup \{k \in \text{Server} : \text{matchIndex}'[i][k] \geq \text{index}\}\]
\[\text{The maximum indexes for which a quorum agrees}\]
\[\text{agreeIndexes} \triangleq \{\text{index} \in 1..\text{Len}(\log[i]) : \text{Agree}(\text{index}) \in \text{Quorum}\}\]

\[\text{New value for commitIndex}'[i]\]
\[\text{newCommitIndex} \triangleq\]
\[\text{IF } \land \text{agreeIndexes} \neq \{\}\]
\[\land \log[i][\text{Max}(\text{agreeIndexes})].\text{term} = \text{currentTerm}[i]\]
\[\text{THEN}\]
\[\text{Max}(\text{agreeIndexes})\]
\[\text{ELSE}\]
\[\text{commitIndex}[i]\]
\[\text{IN } \text{commitIndex}' = [\text{commitIndex} \text{EXCEPT } ![i] = \text{newCommitIndex}]\]
\[\land \text{UNCHANGED} \langle \text{messages}, \text{serverVars}, \text{candidateVars}, \text{leaderVars}, \text{log} \rangle\]

Message handlers
\[i = \text{recipient}, j = \text{sender}, m = \text{message}\]

Server i receives a \text{RequestVote} request from server j with
\[m.mterm \leq \text{currentTerm}[i].\]
APPENDIX B. SAFETY PROOF AND FORMAL SPECIFICATION

\[\text{HandleRequestVoteRequest}(i, j, m) \triangleq\]
\[
\text{LET } \text{logOk} \triangleq \lor m.mlastLogTerm > \text{LastTerm}(log[i])
\]
\[
\lor \land m.mlastLogTerm = \text{LastTerm}(log[i])
\]
\[
\land m.mlastLogIndex \geq \text{Len}(log[i])
\]
\[
\text{grant} \triangleq \land m.mterm = \text{currentTerm}[i]
\]
\[
\land \text{logOk}
\]
\[
\land \text{votedFor}[i] \in \{\text{Nil}, j\}
\]
\[
\text{IN } \land m.mterm \leq \text{currentTerm}[i]
\]
\[
\lor \land \text{grant} \land \text{votedFor}' = [\text{votedFor} \text{EXCEPT} ![i] = j]
\]
\[
\lor \neg \text{grant} \land \text{UNCHANGED votedFor}
\]
\[
\land \text{Reply}(mtype \mapsto \text{RequestVoteResponse},
\]
\[
mterm \mapsto \text{currentTerm}[i],
\]
\[
mvoteGranted \mapsto \text{grant},
\]
\[
\text{mlog is used just for the elections history variable for}
\]
\[
\text{the proof. It would not exist in a real implementation.}
\]
\[
mlog \mapsto \text{log}[i],
\]
\[
msource \mapsto i,
\]
\[
mdest \mapsto j,
\]
\[
m)
\]
\[
\land \text{UNCHANGED } (\text{state}, \text{currentTerm}, \text{candidateVars}, \text{leaderVars}, \text{logVars})
\]

\[\text{Server } i \text{ receives a RequestVote response from server } j \text{ with}\]
\[
m.mterm = \text{currentTerm}[i].\]
\[\text{HandleRequestVoteResponse}(i, j, m) \triangleq\]
\[
\text{This tallies votes even when the current state is not Candidate, but}\]
\[
\land m.mterm = \text{currentTerm}[i]
\]
\[
\land \text{votesResponded}' = [\text{votesResponded} \text{EXCEPT} ![i] =
\]
\[
\text{votesResponded}[i] \cup \{j\}\]
\[
\land \lor \land m.mvoteGranted
\]
\[
\land \text{votesGranted}' = [\text{votesGranted} \text{EXCEPT} ![i] =
\]
\[
\text{votesGranted}[i] \cup \{j\}\]
\[
\land \text{voterLog}' = [\text{voterLog} \text{EXCEPT} ![i] =
\]
\[
\text{voterLog}[i] @@ (j :> m.mlog)]\]
\[ \lor \land \neg m.voteGranted \]
\[ \land \text{UNCHANGED } \langle \text{votesGranted, voterLog} \rangle \]
\[ \land \text{Discard}(m) \]
\[ \land \text{UNCHANGED } \langle \text{serverVars, votedFor, leaderVars, logVars} \rangle \]

Server \(i\) receives an \(\text{AppendEntries}\) request from server \(j\) with 
\[ m.mterm \leq \text{currentTerm}[i] \]
This just handles \(m\).entries of length 0 or 1, but implementations could safely accept more by treating them the same as multiple independent requests of 1 entry.

\(\text{HandleAppendEntriesRequest}(i, j, m) \triangleq\)
\[ \text{LET } \logOk \triangleq \lor \land m.mprevLogIndex = 0 \]
\[ \quad \lor \land m.mprevLogIndex > 0 \]
\[ \quad \land m.mprevLogIndex \leq \text{Len}(\log[i]) \]
\[ \quad \land m.mprevLogTerm = \log[i][m.mprevLogIndex].\text{term} \]
\[ \text{IN } \land m.mterm \leq \text{currentTerm}[i] \]
\[ \land \lor \land \text{reject request} \]
\[ \quad \lor \land m.mterm < \text{currentTerm}[i] \]
\[ \quad \lor \land m.mterm = \text{currentTerm}[i] \]
\[ \quad \land \text{state}[i] = \text{Follower} \]
\[ \land \lnot \logOk \]
\[ \land \text{Reply}(\langle \text{mtype} \mapsto \text{AppendEntriesResponse}, \text{mterm} \mapsto \text{currentTerm}[i], \text{msuccess} \mapsto \text{FALSE}, \text{mmatchIndex} \mapsto 0, \text{msource} \mapsto i, \text{mdest} \mapsto j, m \rangle) \]
\[ \land \text{UNCHANGED } \langle \text{serverVars, logVars} \rangle \]
\[ \lor \text{return to follower state} \]
\[ \land m.mterm = \text{currentTerm}[i] \]
\[ \land \text{state}[i] = \text{Candidate} \]
\[ \land \text{state}' = [\text{state} \text{ EXCEPT } ![i] = \text{Follower}] \]
\[ \land \text{UNCHANGED } \langle \text{currentTerm, votedFor, logVars, messages} \rangle \]
\[ \lor \text{accept request} \]
\[\land m.\text{mterm} = \text{currentTerm}[i]\]
\[\land \text{state}[i] = \text{Follower}\]
\[\land \logOk\]
\[\land \text{LET } \text{index} \xlongequal{\Delta} m.\text{mprevLogIndex} + 1\]
\[\text{IN } \lor \text{ already done with request}\]
\[\land \lor m.\text{mentries} = \langle \rangle\]
\[\lor \land \text{Len}(\log[i]) \geq \text{index}\]
\[\land \log[i][\text{index}].\text{term} = m.\text{mentries}[1].\text{term}\]
\[\text{This could make our } \text{commitIndex} \text{ decrease (for example if we process an old, duplicated request),}
\text{but that doesn’t really affect anything.}\]
\[\land \text{commitIndex}' = [\text{commitIndex} \text{ EXCEPT } ![i] = m.\text{mcommitIndex}]\]
\[\land \text{Reply}(\text{mtype} \mapsto \text{AppendEntriesResponse},\]
\[m\text{term} \mapsto \text{currentTerm}[i],\]
\[m\text{success} \mapsto \text{TRUE},\]
\[m\text{matchIndex} \mapsto m.\text{mprevLogIndex} +\]
\[\text{Len}(m.\text{mentries}),\]
\[m\text{source} \mapsto i,\]
\[m\text{dest} \mapsto j,\]
\[m)\]
\[\land \text{UNCHANGED } \langle \text{serverVars, logVars} \rangle\]
\[\lor \text{conflict: remove 1 entry}\]
\[\land m.\text{mentries} \neq \langle \rangle\]
\[\land \text{Len}(\log[i]) \geq \text{index}\]
\[\land \log[i][\text{index}].\text{term} \neq m.\text{mentries}[1].\text{term}\]
\[\land \text{LET new } \xlongequal{\Delta} [\text{index}2 \in 1 \ldots (\text{Len}(\log[i]) - 1) \mapsto\]
\[\log[i][\text{index}2]\]
\[\text{IN } \log' = [\log \text{ EXCEPT } ![i] = \text{new}]\]
\[\land \text{UNCHANGED } \langle \text{serverVars, commitIndex, messages} \rangle\]
\[\lor \text{no conflict: append entry}\]
\[\land m.\text{mentries} \neq \langle \rangle\]
\[\land \text{Len}(\log[i]) = m.\text{mprevLogIndex}\]
\[ \land \log' = [\log \text{ EXCEPT } ![i] = \text{Append}(\log[i], m.mentries[1])] \]
\[ \land \text{UNCHANGED } (\text{serverVars, commitIndex, messages}) \]
\[ \land \text{UNCHANGED } (\text{candidateVars, leaderVars}) \]

Server i receives an AppendEntries response from server j with
\[ m.mterm = \text{currentTerm}[i]. \]

\[ \begin{align*} 
& \land m.mterm = \text{currentTerm}[i] \\
& \lor \land m.msuccess \ \text{successful} \\
& \land \text{nextIndex}' = [\text{nextIndex EXCEPT } ![i][j] = m.mmatchIndex + 1] \\
& \land \text{matchIndex}' = [\text{matchIndex EXCEPT } ![i][j] = m.mmatchIndex] \\
& \lor \land \neg m.msuccess \ \text{not successful} \\
& \land \text{nextIndex}' = [\text{nextIndex EXCEPT } ![i][j] = \text{Max}(\{\text{nextIndex}[i][j] - 1, 1\})] \\
& \land \text{UNCHANGED } (\text{matchIndex}) \\
& \land \text{Discard}(m) \\
& \land \text{UNCHANGED } (\text{serverVars, candidateVars, logVars, elections}) \\
\end{align*} \]

Any RPC with a newer term causes the recipient to advance its term first.
\[ \text{UpdateTerm}(i, j, m) \triangleq \]
\[ \land m.mterm > \text{currentTerm}[i] \\
\land \text{currentTerm}' = [\text{currentTerm EXCEPT } ![i] = m.mterm] \\
\land \text{state}' = [\text{state EXCEPT } ![i] = \text{Follower}] \\
\land \text{votedFor}' = [\text{votedFor EXCEPT } ![i] = \text{Nil}] \\
\]
messages is unchanged so m can be processed further.
\[ \land \text{UNCHANGED } (\text{messages, candidateVars, leaderVars, logVars}) \]

Responses with stale terms are ignored.
\[ \text{DropStaleResponse}(i, j, m) \triangleq \]
\[ \land m.mterm < \text{currentTerm}[i] \\
\land \text{Discard}(m) \\
\land \text{UNCHANGED } (\text{serverVars, candidateVars, leaderVars, logVars}) \]

Receive a message.
$Receive(m) \triangleq$

\[
\text{LET } i \triangleq m.mdest \quad j \triangleq m.msource
\]

\(\text{IN Any RPC with a newer term causes the recipient to advance its term first. Responses with stale terms are ignored.}
\)

\(\lor UpdateTerm(i, j, m)
\)

\(\lor \land m.mtype = RequestVoteRequest
\)

\(\land HandleRequestVoteRequest(i, j, m)
\)

\(\lor \land m.mtype = RequestVoteResponse
\)

\(\land \lor DropStaleResponse(i, j, m)
\)

\(\lor HandleRequestVoteResponse(i, j, m)
\)

\(\lor \land m.mtype = AppendEntriesRequest
\)

\(\land HandleAppendEntriesRequest(i, j, m)
\)

\(\lor \land m.mtype = AppendEntriesResponse
\)

\(\land \lor DropStaleResponse(i, j, m)
\)

\(\lor HandleAppendEntriesResponse(i, j, m)
\)

End of message handlers.

Network state transitions

The network duplicates a message

$DuplicateMessage(m) \triangleq$

\(\land Send(m)
\)

\(\land \text{UNCHANGED }\langle \text{serverVars, candidateVars, leaderVars, logVars} \rangle
\)

The network drops a message

$DropMessage(m) \triangleq$

\(\land \text{Discard}(m)
\)

\(\land \text{UNCHANGED }\langle \text{serverVars, candidateVars, leaderVars, logVars} \rangle
\)

Defines how the variables may transition.

$Next \triangleq \land \lor \exists i \in \text{Server} : \text{Restart}(i)$

\(\lor \exists i \in \text{Server} : \text{Timeout}(i)$
\[
\begin{align*}
\forall i, j \in \text{Server} : & \, RequestVote(i, j) \\
\forall i \in \text{Server} : & \, \text{BecomeLeader}(i) \\
\forall i \in \text{Server}, v \in \text{Value} : & \, \text{ClientRequest}(i, v) \\
\forall i \in \text{Server} : & \, \text{AdvanceCommitIndex}(i) \\
\forall i, j \in \text{Server} : & \, \text{AppendEntries}(i, j) \\
\forall m \in \text{DOMAIN messages} : & \, \text{Receive}(m) \\
\forall m \in \text{DOMAIN messages} : & \, \text{DuplicateMessage}(m) \\
\forall m \in \text{DOMAIN messages} : & \, \text{DropMessage}(m) \\
\text{History variable that tracks every log ever:} & \\
\land \, \text{allLogs}' = \text{allLogs} \cup \{\log[i] : i \in \text{Server}\}
\end{align*}
\]

The specification must start with the initial state and transition according to \(\text{Next}\).

\[
\text{Spec} \triangleq \text{Init} \land \Box [\text{Next}]_{\text{vars}}
\]

### B.3 Proof

**Lemma 1.** Each server’s currentTerm monotonically increases:

\[
\forall i \in \text{Server} : \\
\text{currentTerm}[i] \leq \text{currentTerm}'[i]
\]

**Proof.** This follows immediately from the specification. \(\square\)

**Lemma 2.** There is at most one leader per term:

\[
\forall e, f \in \text{elections} : \\
ed.\text{eterm} = f.\text{eterm} \Rightarrow e.\text{eleader} = f.\text{eleader}
\]

This is the Election Safety property of Figure 3.2.

**Sketch.** It takes votes from a quorum to become leader, voters may only vote once per term, and any two quorums overlap.

**Proof.**

1. Consider two elections, \(e\) and \(f\), both members of \(\text{elections}\), where \(e.\text{eterm} = f.\text{eterm}\).
2. $e.evotes \in Quorum$ and $f.evotes \in Quorum$, since this is a necessary condition for members of $elections$.

3. Let $voter$ be an arbitrary member of $e.evotes \cap f.evotes$. Such a member must exist since any two quorums overlap.

4. Once $voter$ casts a vote for $e.eleader$ in $e.eterm$, it cannot cast a vote for a different server in $e.eterm$ (the specification ensures this: once it increments its $currentTerm$, it can never vote again for the same server (Lemma 1); and until then, it safely retains its vote information).

5. $e.eleader = f.eleader$, since $voter$ voted for $e.eleader$ and $voter$ voted for $f.eleader$ in $e.eterm = f.eterm$.

\[ \square \]

**Lemma 3.** A leader’s log monotonically grows during its term:

$$\forall e \in elections:\quad currentTerm[e.leader] = e.term \Rightarrow \forall index \in 1..Len(log[e.leader]) : \quad log'[e.leader][index] = log[e.leader][index]$$

This is the Leader Append-Only property of Figure 3.2.

**Sketch.** As a leader, server $i$ only appends to its log; $i$ won’t ever get an AppendEntries request from some other server for the same term, since there is at most one leader per term; and $i$ rejects AppendEntries requests for other terms until increasing its own term.

**Proof.**

1. Three variables are involved in the goal: $elections$, $currentTerm$, and $log$. We consider the transitions that change each of these variables in turn; otherwise, the invariant trivially holds by the inductive hypothesis.

2. When a new election is added to $elections$ (a history variable which maintains information about all successful elections), the $log$ of the leader is not changed in the same step ($log'[e.leader] = log[e.leader]$), so the invariant is maintained.

3. $currentTerm[e.leader]$ monotonically increases by Lemma 1, so once $e.leader$ moves to a new term, it will trivially satisfy the invariant forever after.
4. *log* changes either from client requests or AppendEntries requests:
   
   (a) Case: client request:
   
   i. By the specification, the leader only appends an entry to its log, which maintains
   the invariant.

   (b) Case: AppendEntries request:
   
   i. Only servers with \( \text{state}[i] = \text{Leader} \) can send AppendEntries requests for their
   currentTerm.
   
   ii. By Lemma 2, \( \text{e.leader} \) is the only server which can ever be leader for \( \text{e.term} \).
   
   iii. Servers don’t send themselves AppendEntries requests (see specification).
   
   iv. \( \text{e.leader} \) will process no AppendEntries requests while its term is \( \text{e.term} \).

\[ \text{Lemma 4.} \] An \( \langle \text{index}, \text{term} \rangle \) pair identifies a log prefix:

\[ \forall l, m \in \text{allLogs} : \]
\[ \forall \langle \text{index}, \text{term} \rangle \in l : \]
\[ \langle \text{index}, \text{term} \rangle \in m \Rightarrow \]
\[ \forall \text{pindex} \in 1..\text{index} : \]
\[ l[\text{pindex}] = m[\text{pindex}] \]

This is the Log Matching property of Figure 3.2.

**Sketch.** Only leaders create entries, and they assign the new entries term numbers that will never
be assigned again by other leaders (there’s at most one leader per term). Moreover, the consistency
check in AppendEntries guarantees that when followers accept new entries, they do so in a way
that’s consistent with the leader’s log at the time it sent the entries.

**Assertion.** If \( p \) is a prefix of some log \( l \in \text{allLogs} \), then \( \text{allLogs}' = \text{allLogs} \cup \{ p \} \) maintains the
invariant (the statement in the lemma).

1. This follows immediately from the invariant, since \( p \)’s entries match \( l \)’s entries, and \( p \) con-
tributes no additional entries.

**Proof by induction on an execution.**
1. Initial state: all of the servers’ logs are empty, so \( \text{allLogs} = \emptyset \), and the invariant trivially holds.

2. Inductive step: logs change in one of the following ways:

   (a) Case: a leader adds one entry (client request)
      
      i. By the inductive hypothesis, \( \text{log[leader]} \in \text{allLogs} \).
      
      ii. The \( \langle \text{index}, \text{term} \rangle \) of the new entry cannot exist in any other entry in any log in \( \text{allLogs} \), since there’s only one leader per term (Lemma 2) and leaders only append to their logs (Lemma 3).
      
      iii. Then \( \text{allLogs}' = \text{allLogs} \cup \{ \text{log[leader]} \| \langle \text{index}, \text{term} \rangle \} \) maintains the invariant.

   (b) Case: a follower removes one entry (AppendEntries request \( m \))
      
      i. The invariant still holds, since \( \text{log'[follower]} \) is a prefix of \( \text{log[follower]} \) (by the Assertion above).

   (c) Case: a follower adds one entry (AppendEntries request \( m \))
      
      i. \( m.m\log \) is a copy of the leader’s log at the time the leader created the AppendEntries request.
      
      ii. \( m.m\log \in \text{allLogs} \) by definition of \( \text{allLogs} \).
      
      iii. In the two cases below, we show that \( \text{log'[follower]} \) is a prefix of \( m.m\log \).
      
      iv. Case: \( m.m\text{prevLogIndex} = 0 \)
          
          A. \( m.m\text{entries} \) is a prefix of \( m.m\log \).
          
          B. \( \text{log[follower]} \) is empty, as a necessary condition for accepting the request (the specification separates transitions for removing a conflicting entry, replying when there is no longer any change to make, and appending an entry).
          
          C. \( \text{log'[follower]} = m.m\text{entries} \) upon accepting the request, which is a prefix of \( m.m\log \).
      
      v. Case: \( m.m\text{prevLogIndex} > 0 \)
          
          A. \( \text{start} \| \langle m.m\text{prevLogIndex}, m.m\text{prevLogTerm} \rangle \| m.m\text{entries} \) is a prefix of \( m.m\log \), where \( \text{start} \) is some (possibly empty) log prefix.
          
          B. The follower accepts the request by assumption, so its log contains the entry \( \langle m.m\text{prevLogIndex}, m.m\text{prevLogTerm} \rangle \).
C. By the inductive hypothesis, \( \text{log}[\text{follower}] \) contains the prefix 
\( \text{start} \| \langle m.\text{mprevLogIndex}, m.\text{mprevLogTerm} \rangle \).

D. \( \text{log}'[\text{follower}] = \text{start} \| \langle m.\text{mprevLogIndex}, m.\text{mprevLogTerm} \rangle \| m.\text{entries} \)
upon accepting the request, which is a prefix of \( m.\text{mlog} \).

vi. Because \( \text{log}'[\text{follower}] \) is a prefix of \( m.\text{mlog} \), the invariant is maintained (by the
Assertion above).

Lemma 5. When a follower appends an entry to its log, its log after the append is a prefix of the
leader’s log at the time the leader sent the AppendEntries request:

\[
\forall i \in \text{Server} : \\
\text{state}[i] \neq \text{Leader} \land \text{Len}(\text{log}'[i]) > \text{Len}(\text{log}[i]) \implies \\
\exists m \in \text{DOMAIN messages} : \\
\land m.\text{mtype} = \text{AppendEntriesRequest} \\
\land \forall index \in 1..\text{Len}(\text{log}'[i]) : \\
\text{log}'[i][index] = m.\text{mlog}[index]
\]

This restates an argument from the proof of Lemma 4 that is useful in the proofs of other lemmas.
(The argument is difficult to make before Lemma 4, since that lemma’s inductive hypothesis is key;
however, the proof for this lemma follows easily from Lemma 4.)

Sketch. The new entry that the follower appends to its log was also present in the leader’s log. Thus,
by Lemma 4, the follower’s new log is a prefix of what was the leader’s log.

Proof. Logs change in one of the following ways:

1. Case: a leader adds one entry (client request). This invariant only applies to non-leaders.

2. Case: a follower removes one entry (AppendEntries request). This invariant only affects logs
that grow in length.

3. Case: a follower adds one entry (AppendEntries request \( m \)):

   (a) \( m.\text{mlog} \) is a copy of the leader’s log at the time the leader created the AppendEntries
   request.

   (b) Thus, \( m.\text{mlog} \in \text{allLogs} \).
(c) $log'[i] \in allLogs$ by definition of $allLogs$.

(d) $m.mentries$, the entry being added, is the last entry in $log'[i]$. (This extends to multiple entries for implementations that batch entries together.)

(e) $m.mentries \in m.mlog$

(f) By Lemma 4, the index and term of $m.mentries$ uniquely identifies a prefix of $m.mlog$ equal to $log'[i]$.

Lemma 6. A server’s current term is always at least as large as the terms in its log:

$$\forall i \in Server :$$
$$\forall (index, term) \in log[i] :$$
$$term \leq currentTerm[i]$$

Sketch. Servers’ current terms monotonically increase. When leaders create new entries, they assign them their current term. And when followers accept new entries from a leader, their current term agrees with the leader’s term at the time it sent the entries.

Proof by induction on an execution.

1. Initial state: all logs are empty, so the invariant trivially holds.

2. Inductive step: $currentTerm[i]$ changes:
   (a) By Lemma 1, $currentTerm'[i] \geq currentTerm[i]$, so the invariant is maintained.

3. Inductive step: logs change in one of the following ways:
   (a) Case: a leader adds one entry (client request):
      (i) By the inductive hypothesis, all entries in $log[i]$ have $term \leq currentTerm[i]$.
      (ii) The new entry’s term is $currentTerm[i]$.
      (iii) Thus, all entries in $log'[i]$ satisfy the invariant.
   (b) Case: a follower removes one entry (AppendEntries request)
      (i) The invariant still holds, since only the length of the log decreased.
   (c) Case: a follower adds one entry (AppendEntries request $m$):
i. By the inductive hypothesis, when the leader created the request, its current term was at least as large as the term of every entry in its log:
   \[ \forall \langle \text{index}, \text{term} \rangle \in m.mlog \colon \text{term} \leq m.mterm \]

ii. \( \log'[i] \) is a prefix of \( m.mlog \) by Lemma 5.

iii. As a necessary condition for accepting the request, \( \text{currentTerm}[i] = m.mterm. \)

iv. Then \( \text{currentTerm}[i] \) is at least as large as the term in every entry in \( \log'[i] \), and the invariant is maintained.

Lemma 7. The terms of entries grow monotonically in each log:

\[ \forall l \in \text{allLogs} : \]
\[ \forall \text{index} \in 1..(\text{Len}(l) - 1) : \]
\[ l[\text{index}].\text{term} \leq l[\text{index} + 1].\text{term} \]

Sketch. A leader maintains this by assigning new entries its current term, which is always at least as large as the terms in its log. When followers accept new entries, they are consistent with the leader’s log at the time it sent the entries.

Proof by induction on an execution.

1. Initial state: all logs are empty, so the invariant holds.

2. Inductive step: logs change in one of the following ways:

   (a) Case: a leader adds one entry (client request)
      i. The new entry’s term is \( \text{currentTerm}[\text{leader}] \)
      ii. \( \text{currentTerm}[\text{leader}] \) is at least as large as the term of any entry in \( \log[\text{leader}] \), by Lemma 6.

   (b) Case: a follower removes one entry (AppendEntries request)
      i. The invariant still holds, since only the length of the log decreased.

   (c) Case: a follower adds one entry (AppendEntries request \( m \))
      i. \( \log'[\text{follower}] \) is a prefix of \( m.mlog \) (by Lemma 5).
      ii. \( m.mlog \in \text{allLogs} \)
iii. By the inductive hypothesis, the terms in $m. mlog$ monotonically grow, so the terms in $log'[follower]$ monotonically grow.

**Definition 1.** An entry $\langle index, term \rangle$ is **committed at term** $t$ if it is present in every leader’s log following $t$:

$$\text{committed}(t) \triangleq \{ \langle index, term \rangle : \forall election \in \text{elections} :$$

$$\text{election.eterm} > t \Rightarrow$$

$$\langle index, term \rangle \in \text{election.elog} \}$$

**Definition 2.** An entry $\langle index, term \rangle$ is **immediately committed** if it is acknowledged by a quorum (including the leader) during $term$. Lemma 8 shows that these entries are committed at $term$.

$$\text{immediatelyCommitted} \triangleq \{ \langle index, term \rangle \in \text{anyLog} :$$

$$\land \text{anyLog} \in \text{allLogs}$$

$$\land \exists \text{leader} \in \text{Server}, \text{subquorum} \in \text{SUBSET Server} :$$

$$\land \text{subquorum} \cup \{ \text{leader} \} \in \text{Quorum}$$

$$\land \forall i \in \text{subquorum} :$$

$$\exists m \in \text{messages} :$$

$$\land m.mtype = \text{AppendEntriesResponse}$$

$$\land m.msouce = i$$

$$\land m.mdest = \text{leader}$$

$$\land m.mterm = \text{term}$$

$$\land m.mmatchIndex \geq index \}$$

**Lemma 8.** Immediately committed entries are committed:

$$\forall \langle index, term \rangle \in \text{immediatelyCommitted} :$$

$$\langle index, term \rangle \in \text{committed}(\text{term})$$

Along with Lemma 9, this is the Leader Completeness property of Figure 3.2.

**Sketch.** See Section 3.6.3.

**Proof.**
1. Consider an entry \((index, term)\) that is immediately committed.

2. Define

\[
Contradicting \triangleq \{election \in elections: \\
\land election.eterm > term \\
\land (index, term) \notin election.elog\}
\]

3. Let \(election\) be an element in \(Contradicting\) with a minimal \(term\) field. That is,
\[
\forall e \in Contradicting: election.eterm \leq e.eterm.
\]
If more than one election has the same term, choose the earliest one. (The specification does not allow this to happen, but it is safe for a leader to step down and become leader again in the same term.)

4. It suffices to show a contradiction, which implies \(Contradicting = \emptyset\).

5. Let \(voter\) be any server that both votes in \(election\) and contains \((index, term)\) in its log during \(term\) (either it acknowledges the entry as a follower or it was leader). Such a server must exist since:

(a) A quorum of servers voted in \(election\) for it to succeed.

(b) A quorum contains \((index, term)\) in its log during \(term\), since \((index, term)\) is immediately committed.

(c) Any two quorums overlap.

6. Let \(voterLog \triangleq election.evoterLog[voter]\), the voter’s log at the time it cast its vote.

7. The voter contains the entry when it cast its vote during \(election.eterm\). That is, \((index, term) \in voterLog:\)

(a) \((index, term)\) was in the voter’s log during \(term\).

(b) The voter must have stored the entry in \(term\) before voting in \(election.eterm\), since:
\[\]
   i. \(election.eterm > term\).
\[\]
   ii. The voter rejects requests with terms smaller than its current term, and its current term monotonically increases (Lemma 1).

(c) The voter couldn’t have removed the entry before casting its vote:
i. Case: No $\text{AppendEntriesRequest}$ with $m\text{term} < \text{term}$ removes the entry from the voter’s log, since $\text{currentTerm}[\text{voter}] \geq \text{term}$ upon storing the entry (by Lemma 6), and the voter rejects requests with terms smaller than $\text{currentTerm}[\text{voter}]$.

ii. Case: No $\text{AppendEntriesRequest}$ with $m\text{term} = \text{term}$ removes the entry from the voter’s log, since:
   A. There is only one leader of $\text{term}$.
   B. The leader of $\text{term}$ created and therefore contains the entry (Lemma 3).
   C. The leader would not send any conflicting requests to $\text{voter}$ during $\text{term}$.

iii. Case: No $\text{AppendEntriesRequest}$ with $m\text{term} > \text{term}$ removes the entry from the voter’s log, since:
   A. Case: $m\text{term} > \text{election.eterm}$:
      This can’t happen, since $\text{currentTerm}[\text{voter}] > \text{election.eterm}$ would have prevented the voter from voting in $\text{term}$.
   B. Case: $m\text{term} = \text{election.eterm}$:
      Since there is at most one leader per term (Lemma 2), this request would have to come from $\text{election.eleader}$ as a result of an earlier election in the same term ($\text{election.eterm}$).
      Because a leader’s log grows monotonically during its term (by Lemma 3), the leader could not have had $(\text{index, term})$ in its log at the start of its term.
      Then there exists an earlier election with the same term in $\text{Contradicting}$; this is a contradiction.
   C. Case $m\text{term} < \text{election.eterm}$:
      The leader of $m\text{term}$ must have contained the entry (otherwise its election would also be $\text{Contradicting}$ but have a smaller term than $\text{election}$, which is a contradiction). Thus, the leader of $m\text{term}$ could not send any conflicting entries to the voter for this index, nor could it send any conflicting entries for prior indexes: that it has this entry implies that it has the entire prefix before it (Lemma 4).

8. The log comparison during elections states the following, since $\text{voter}$ granted its vote during $\text{election}$:

\[ \lor \text{LastTerm}(\text{election.elog}) > \text{LastTerm}(\text{voterLog}) \]
\[ \begin{align*} &\vee \land \text{LastTerm}(\text{election.elog}) = \text{LastTerm}(\text{voterLog}) \\ &\land \text{Len}(\text{election.elog}) \geq \text{Len}(\text{voterLog}) \end{align*} \]

In the following two steps, we take each of these cases in turn and show a contradiction.

9. Case: \( \text{LastTerm}(\text{election.elog}) = \text{LastTerm}(\text{voterLog}) \) and \( \text{Len}(\text{election.elog}) \geq \text{Len}(\text{voterLog}) \)

   (a) The leader of \( \text{LastTerm}(\text{voterLog}) \) monotonically grew its log during its term (by Lemma 3).

   (b) The same leader must have had \( \text{election.elog} \) as its log at some point, since it created the last entry.

   (c) Thus, \( \text{voterLog} \) is a prefix of \( \text{election.elog} \).

   (d) Then \( (\text{index}, \text{term}) \in \text{election.elog} \), since \( (\text{index}, \text{term}) \in \text{voterLog} \).

   (e) But \( \text{election} \in \text{Contradicting} \) implies that \( (\text{index}, \text{term}) \notin \text{election.elog} \).

10. Case: \( \text{LastTerm}(\text{election.elog}) > \text{LastTerm}(\text{voterLog}) \)

   (a) \( \text{LastTerm}(\text{voterLog}) \geq \text{term} \), since \( (\text{index}, \text{term}) \in \text{voterLog} \) and terms in logs grow monotonically (Lemma 7).

   (b) \( \text{election.eterm} > \text{LastTerm}(\text{election.elog}) \) since servers increment their \( \text{currentTerm} \) when starting an election, and Lemma 6 states that a server’s \( \text{currentTerm} \) is at least as large as the terms in its log.

   (c) Let \( \text{prior} \) be the election in \( \text{elections} \) with \( \text{prior.eterm} = \text{LastTerm}(\text{election.elog}) \). Such an election must exist since \( \text{LastTerm}(\text{election.elog}) > 0 \) and a server must win an election before creating an entry.

   (d) By transitivity, we now have the following inequalities:

   \[ \begin{align*} \text{term} \leq & \text{LastTerm}(\text{voterLog}) < \\ & \text{LastTerm}(\text{election.elog}) = \text{prior.eterm} < \\ & \text{election.eterm} \end{align*} \]

   (e) \( (\text{index}, \text{term}) \in \text{prior.elog} \), since \( \text{prior} \notin \text{Contradicting} \) (\( \text{election} \) was assumed to have the lowest term of any election in \( \text{Contradicting} \), and \( \text{prior.eterm} < \text{election.eterm} \)).
(f) $\text{prior.elog}$ is a prefix of $\text{election.elog}$ since:

i. $\text{prior.eleader}$ creates entries with $\text{prior.eterm}$ by appending them to its log, which monotonically grows during $\text{prior.eterm}$ from $\text{prior.elog}$.

ii. Thus, any entry with term $\text{prior.eterm}$ must follow $\text{prior.elog}$ in all logs (by Lemma 4).

iii. $\text{LastTerm(election.elog)} = \text{prior.eterm}$

(g) $\langle \text{index}, \text{term} \rangle \in \text{election.elog}$

(h) This is a contradiction, since $\text{election.elog}$ was assumed to not contain the committed entry ($\text{election} \in \text{Contradicting}$).

\[\square\]

**Definition 3.** An entry $\langle \text{index}, \text{term} \rangle$ is **prefix committed at term** $t$ if there is another entry that is committed at term $t$ following it in some log. Lemma 9 shows that these entries are committed at term $t$.

\[
\text{prefixCommitted}(t) \equiv \{ \langle \text{index}, \text{term} \rangle \in \text{anyLog} : \neg \exists \langle \text{rindex}, \text{rterm} \rangle \in \text{anyLog} : \langle \text{index}, \text{term} \rangle \notin \text{committed}(t) \}
\]

**Lemma 9.** Prefix committed entries are committed in the same term:

\[\forall t : \text{prefixCommitted}(t) \subseteq \text{committed}(t)\]

Along with Lemma 8, this is the Leader Completeness property of Figure 3.2.

**Sketch.** If an entry is committed, it identifies a prefix of a log in which every entry is committed, since those entries will also be present in every future leader’s log.

**Proof.**

1. Consider an arbitrary entry $\langle \text{index}, \text{term} \rangle \in \text{prefixCommitted}(t)$.

2. There exists an entry $\langle \text{rindex}, \text{rterm} \rangle \in \text{committed}(t)$ following $\langle \text{index}, \text{term} \rangle$ in some log, by definition of $\text{prefixCommitted}(t)$.
3. \(\langle rindex, rterm \rangle\) uniquely identifies the log prefix containing \(\langle index, term \rangle\) (Lemma 4).

4. Every leader following \(t\) contains \(\langle index, term \rangle\), since every leader following \(t\) contains \(\langle rindex, rterm \rangle\).

5. \(\langle index, term \rangle \in committed(t)\) by definition of \(committed(t)\).

\(\square\)

**Theorem 1.** Servers only apply entries that are committed in their current term:

\[
\forall i \in Server : \quad \land \text{commitIndex}[i] \leq \text{Len}(\log[i]) \\
\land \forall \langle index, term \rangle \in \log[i] : \\
\quad \text{index} \leq \text{commitIndex}[i] \Rightarrow \\
\quad \langle index, term \rangle \in committed(\text{currentTerm}[i])
\]

This is equivalent to the State Machine Safety property of Figure 3.2. (The bound on the commit index is needed to strengthen the inductive hypothesis.)

**Sketch.** A leader only advances its commitIndex to cover entries that are immediately committed or prefix committed. Followers update their commitIndex from the leader’s only when they have a prefix of the leader’s log.

**Proof by induction on an execution.**

1. Initial state: trivially holds for empty logs (and commitIndex\([i]\) is initialized to 0).

2. Inductive step: the set of entries committed at currentTerm\([i]\) changes:

   (a) Once an entry is committed at currentTerm\([i]\), all leaders of subsequent terms will have the entry (by the definition of committed).

   (b) Thus, the set of committed entries at currentTerm\([i]\) monotonically grows.

3. Inductive step: commitIndex\([i]\) changes:

   (a) When commitIndex\([i]\) decreases (if implementations allow this to happen), the inductive hypothesis suffices to show the invariant holds.

   (b) When commitIndex\([i]\) increases, it covers entries present in \(i\)’s log that are committed:
i. Case: follower completes accepting AppendEntries request $m$:
   A. Upon processing the request, the follower’s log is a prefix of a prior version of
the leader’s log, $m.mlog$ (by Lemma 5).
   B. Every entry up through $commitIndex'[i]$ in $m.mlog$ is committed by the inductive hypothesis (they were marked committed in the leader’s log when it sent the request).

ii. Case: leader $i$ processes AppendEntries response:
   A. If the leader sets a new $commitIndex$., the conditions in the specification ensure
that $commitIndex'[i] \in immediatelyCommitted$.
   B. Every entry in the leader’s log with index up to $commitIndex'[i]$ is prefix committed at $currentTerm[i]$.

4. Inductive step: $currentTerm[i]$ changes:
   (a) By Lemma 1, $currentTerm'[i] > currentTerm[i]$.
   (b) $committed(currentTerm[i]) \subseteq committed(currentTerm'[i])$ by the definition of $committed$.
   (c) Thus, the inductive hypothesis suffices to show the invariant holds.

5. Inductive step: logs change in one of the following ways:
   (a) Case: a leader adds one entry (client request):
      i. Newly created entries are not marked committed, so the invariant holds.
   (b) Case: a follower removes one entry (AppendEntries request $m$):
      i. Case: the removed entry was not marked committed on the follower:
         The inductive hypothesis suffices to show the invariant holds.
      ii. Case: the removed entry was marked committed on the follower:
         A. $m.mterm = currentTerm[i]$, since the follower accepted the request.
         B. The removed entry is not in $m.mlog$, since it conflicts with the request.
         C. The removed entry is not present in $m.msource$’s log at the start of its term (by Lemma 3).
         D. The election for $m.mterm$ did not contain the removed entry.
         E. The removed entry is not committed at $currentTerm[i]$. 
F. This contradicts the inductive hypothesis; this case cannot occur.

(c) Case: a follower adds one entry (AppendEntries request \( m \)):

i. Case: the new entry is not marked committed on the follower:
   The inductive hypothesis suffices to show the invariant holds.

ii. Case: the new entry is marked committed on the follower:
   \( commitIndex[i] \) must increase (which was already handled above).
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